應數一線性代數 **2019** 秋, 第一次期中考

學號: ———————————————————, 姓名: ———————————————————

本次考試共有 8 頁 (包含封面)，有 12 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知**:**

- 請在第一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。

- 不可翻閱課本或筆記。

- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：誠敬弘遠

誠，一生動念都是誠實端正的。敬，就是對知識的認真尊重。宏，開拓視界，恢宏心胸。遠，任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ———————————————————

以下由閱卷人員填寫

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	10	5	10	10	10	10	10	5	5	10	10	5	100
Score:													

1. (10 points) Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 3 & 1 \end{bmatrix}$$

Compute the following matrices or write DNE if the it is undefined.

  (a) AB

  (b) BA.

  (c) $A + 2B$.

2. (5 points) Determine if the set $W = \{(x, y, z) \in \mathbb{R}^3 | y = x, z = 2x\}$ is a subspace of $\mathbb{R}^3$

3. (10 points) Consider the following linear system $\begin{cases} x_1 - x_2 + x_3 + x_4 = 5 \\ x_2 - x_3 + 2x_4 = 8 \\ 2x_1 - x_2 - 3x_3 + 4x_4 = 18 \end{cases}$

    (a) Write down the corresponding augmented matrix and reduce it to row-echelon form.

    (b) Reduce the augmented matrix further to reduced row-echelon form.

    (c) Write down the solution of the original linear system.

4. (10 points) Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

(a) Compute the inverse of A and verify that you have the correct inverse.

(b) Use part (a) to solve

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

5. (10 points) Is the following set of vectors dependent or independent?

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} \right\}$$

6. (10 points) Find a basis for (a) the nullspace, (b) the column space, and (c) the row space of the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 4 \\ 1 & 2 & 1 & 1 & 6 \\ 0 & 1 & 1 & 1 & 3 \\ 2 & 2 & 0 & 1 & 7 \end{bmatrix}$$

7. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T([1, 0, 0]) = [1, 2, 1], T([0, 1, 0]) = [3, 0, 4]$, and $T([1, 0, 1]) = [5, 4, 6]$.

   (a) Find the standard matrix representation of T.

   (b) Use the standard matrix representation to find a formula for $T([x_1, x_2, x_3])$.

   (c) Find the kernel of T.

(d) Is the linear transformation T invertible? If so, find the standard matrix representation of T^{-1}.

8. (5 points) Suppose that T is a linear transformation with standard matrix representation A, and that A is a 7×6 matrix such that the nullspace of A has dimension 4. What is the dimension of the range of T?

9. (5 points) If a 7×9 matrix A has rank 5, find the dimension of the column space of A, the dimension of the nullspace of A, and the dimension of the row space of A.

10. (10 points) Suppose that the vectors $\vec{v}, \vec{w}$, and $\vec{x}$ are mutually perpendicular (i.e. $\vec{v}$ and $\vec{w}$ are perpendicular, $\vec{v}$ and $\vec{x}$ are perpendicular, and $\vec{w}$ and $\vec{x}$ are perpendicular). Use dot products to find $\|v + 3w + 2x\|$ in terms of the magnitudes (lengths) of $\vec{v}, \vec{w}$, and $\vec{x}$. Hint: Start by computing $\|\vec{v} + 3\vec{w} + 2\vec{x}\|^2$.

11. (10 points) In the following transformation, express the standard matrix representation of the given invertible transformation of $\mathbb{R}^2$ into itself as a product of elementary matrices. Use this expression to describe the transformation as a product of one or more reflections, horizontal or vertical expansions or contractions, and shears.

   (a) $T(x, y) = [-y, x]$. (Rotation counter-clockwise through $\frac{\pi}{2}$)

   (b) $T(x, y) = [-x, -y]$. (Rotation through π)

12. (5 points) Classify each of the following statements as True or False. No explanation is necessary.

   (a) _____ If A is a 2×3 matrix and B is a 2×4 matrix, then AB is a 3×4 matrix.

   (b) _____ Any six vectors in $\mathbb{R}^4$ must span $\mathbb{R}^4$.

   (c) _____ Every independent subset of $\mathbb{R}^n$ is a subset of some basis for $\mathbb{R}^n$.

   (d) _____ If A is a 7×4 matrix, and if the dimension of the column space of A is 3,then the columns of A are linearly dependent.

   (e) _____ If T is a linear transformation, then $T(0) = 0$.