**應數一線性代數 2019 秋, 第二次期中考**

學號: ________________________, 姓名: ________________________

本次考試共有 ?? 頁 (包含封面)，有 ?? 題。如有缺頁或漏題，請立刻告知監考人員。

## 考試須知:

- 請在第一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬弘遠**

**誠**，一生動念都是誠實端正的。**敬**，就是對知識的認真尊重。**宏**，開拓視界，恢宏心胸。**遠**，任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ________________________

**以下由閱卷人員填寫**

Run LaTeX again to produce the table

1. (10 points) Consider the set $\mathbb{R}^2$, with the addition defined by $[x, y] \oplus [a, b] = [x + a - 1, y + b]$, and with scalar multiplication defined by $r[x, y] = [rx - r + 1, ry]$. Is this set a vector space?

   If so, $\vec{0} =$ ____________________, $-[x, y] =$ ____________________.

2. (5 points) Consider the set of all polynomials of degree 6 together with the zero polynomial in the vector space $P$ of all polynomials in $x$. Determine whether the given subset is a subspace of the given vector space.

3. (10 points) Determine whether the given set of vector is dependent of independent, and find a basis for it.

   (a) $\{x^2 - 1, x^2 + 1, 4x, 2x + 5\}$

   (b) $\{1, e^{2x} + e^{-2x}, e^{2x} - e^{-2x}\}$.

4. (5 points) Find $\mathbf{v}_B$, which is the coordinate vector of the given vector $\mathbf{v} = [1, 2, -2]$ relative to the indicated ordered basis $\mathbf{B} = \{[1, 1, 1], [1, 2, 0], [1, 0, 1]\}$.

5. (10 points) Determine whether the given set is a vector space of all functions mapping $\mathbb{R}$ into $\mathbb{R}$.

   (a) $\{f|f(0) = 1\}$

   (b) $\{f|f(1) = 0\}$

6. (10 points) Find the coordinate vector of polynomial $4x^3 + 9x^2 + x$ relative to the ordered basis $B = ((x-1)^3, (x-1)^2, (x-1), 1)$ of the vector space $P_3$ of polynomials of degree at most 3.

7. (10 points) Find the polynomial in $P_2$ whose coordinate vector relative to the ordered basis $B = (x + x^2, x - x^2, 1 + x)$ is $[3, 1, 2]$.

8. (10 points) Let $V$ and $V''$ be vector spaces with ordered bases $B = (\vec{b_1}, \vec{b_2}, \vec{b_3})$ and $B' = (\vec{b_1'}, \vec{b_2'}, \vec{b_3'}, \vec{b_4'})$, respectively, and let $T : V \longrightarrow V'$ be the linear transformation having the given matrix $A$ as matrix representation relative to $B, B'$. Find $T(\vec{v})$ for the given vector $\vec{v}$.

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 2 & 0 \\ 0 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix}, \quad \vec{v} = 2\vec{b_1} + \vec{b_2} + 3\vec{b_3}$$

9. (10 points) Let $T : P_3 \longrightarrow P_2$ be defined by $T(p(x)) = D(p(x+1))$, and let $B = (x^3, x^2, x, 1)$ and $B' = (x^2, x, 1)$.

   (a) Find the matrix A as matrix representation of $T$ relative to $B, B'$.

   (b) Use $A$ to compute $T(4x^3 - 3X^2 + 5x - 2)$.

10. (10 points) Let $V = sp(e^{2x}, e^{4x}, e^{8x})$, $V' = sp(e^{3x}, e^{7x}, e^{9x})$ are the subspaces of the vector space of all real-valued functions with domain $\mathbb{R}$, and let $B = (e^{2x}, e^{4x}, e^{8x})$, $B' = (e^{3x}, e^{7x}, e^{9x})$. Let $T : V \longrightarrow V'$ be the linear transformation having the given matrix $A$ as matrix representation relative to $B, B'$.

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix},$$

Find $T(ae^{2x} + be^{4x} + ce^{8x}) =$____________________

11. (10 points) Circle True or False. Read each statement in original Greek before answering.

(a) True False Every vector space contains at least one vector.

(b) True False Every vector space contains at least two vectors.

(c) True False Any two bases in a finite-dimensional vector space V have the same number of elements.

(d) True False Multiplication of two scalars is of no concern to the definition of a vector space.

(e) True False If $\{v_1, v_2, ..., v_n\}$ generates $V$, then each $v \in V$ is a unique linear combination of vectors in this set.