應數一線性代數 2019 秋, 期末考

學號: ______________________, 姓名: ______________________

本次考試共有 9 頁 (包含封面)，有 15 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬弘遠**

誠，一生動念都是誠實端正的。**敬**，就是對知識的認真尊重。**宏**，開拓視界，恢宏心胸。**遠**，任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ______________________

以下由閱卷人員填寫

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	10	10	10	10	10	10	10	100
Score:											

Question:	11	12	13	14	15	Total
Points:	10	10	10	10	10	50
Score:						

1. (10 points) Find the determinant of

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 3 & -1 & 2 & 0 & 0 \\ 0 & 4 & 1 & -1 & 2 \\ 0 & 0 & -3 & 2 & 4 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix}$$

Answer: ________________

2. (10 points) Suppose that A is a 4×4 matrix with determinant 7.

(a) Find $\det(3A) =$ ________________

(b) Find $\det(A^{-1}) =$ ________________

(c) Find $\det(2A^{-1}) =$ ________________

(d) Find $\det((2A)^{-1}) =$ ________________

3. (10 points) Suppose that A is a 3×3 matrix with row vectors $\vec{a}, \vec{b}$, and $\vec{c}$, and that $det(A) = 3$. Find the determinant of the matrix having the indicated row vectors

   (a) $\vec{a}, \vec{b}, 2\vec{a} + 3\vec{b}$.
   Determinant = ________________

   (b) $\vec{a} + 2\vec{a}, \vec{a} + 3\vec{b}, 5\vec{a} + \vec{c}$.
   Determinant = ________________

4. (10 points)

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 4 \\ 1 & 2 & 1 \end{bmatrix}$$

The inverse of A = ________________ , and the adjoint matrix of A = ________________

5. (10 points) Let $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$, $\vec{b} = 4\vec{i} - \vec{j} + 2\vec{k}$, $\vec{c} = 3\vec{i} + \vec{k}$.

(a) $\vec{a} \cdot (\vec{b} \times \vec{c}) =$ ________________

(b) $\vec{a} \times (\vec{b} \times \vec{c}) =$ ________________

6. (10 points) Find out whether points (0,0,0), (1,4,3), (2,5,8) and (-1,2,-5) lie in a plane in $\mathbb{R}^3$

Answer: ________________

7. (10 points) Using Cramer's rule to find the component x_2 of the solution vector for the given linear system.

$$\begin{cases} x + 2y - z = -2 \\ 2x + y + z = 0 \\ 3x - y + 5z = 1 \end{cases}$$

$x_2 =$ ________________

8. (10 points) Find the volume of the n-box in $\mathbb{R}^4$ determined by the vertices(頂點) (1, 0, 0, 1), (-1, 2, 0, 1), (3, 0, 1, 1), (-1, 4, 0, 1)

Answer: ________________

9. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation defined by

$$T([x, y, z]) = [x - 2y, 3x + z, 4x + 3y].$$

Find the volume of the image under T of the ball B $= \{(x, y, z) | x^2 + (y - 3)^2 + (z + 2)^2 \leq 16\}$

Answer: ________________

10. (10 points) Circle True or False. Read each statement in original Greek before answering.

(a) True False The determinant of an upper-triangular(上三角) square matrix is the product o the entires(元素) on its main diagonal.(主對角線)

(b) True False There is no square matrix A such that $\det(A^T A) = -1$.

(c) True False If the image under a linear transformation T of an n-box B in $\mathbb{R}^n$ has volume 15, the box B has volume $|\det(A)| \cdot 15$, if the standard matrix representation of T is A.

(d) True False If $\det(A) = 2, \det(B) = 3$, then $\det(A + B) = 5$.

(e) True False The box in $\mathbb{R}^3$ determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is a cube(正方體) if and only if $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c} = 0$ and $\vec{a} \cdot \vec{a} = \vec{b} \cdot \vec{b} = \vec{c} \cdot \vec{c} = 0$

第二部份，額外加分題

11. (10 points) Let V and V'' be vector spaces with ordered bases $B = ([1,3,-2],[4,1,3],[-1,2,0])$ and $B' = ([1,0,1,0],[2,1,1,-1],[0,1,1,-1],[2,0,3,1])$, respectively, and let $T : V \longrightarrow V'$ be the linear transformation having the given matrix A as matrix representation relative to B, B'. Find $T([3,13,-1])$ for the given vector .

$$A = \begin{bmatrix} 0 & 4 & -1 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix},$$

$T([3,13,-1]) =$ ________________

12. (10 points) Find a basis for (a) the nullspace, (b) the column space, and (c) the row space of the following matrix:

$$A = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 1 & 3 & 0 & 5 \\ 1 & 1 & 6 & 3 \end{bmatrix}$$

13. (10 points) Determine whether the given set of vector is dependent of independent, and find a basis for it.
$\{[1, 1, 1], [2, 3, 1], [2, 1, 3], [1, 0, 2]\}$

14. (10 points) Consider the following linear system $\begin{cases} 2x_1 + x_2 + 3x_3 = 5 \\ x_1 - x_2 + 2x_3 + x_4 = 0 \\ 4x_1 - x_2 + 7x_3 + 2x_4 = 5 \\ -x_1 - 2x_2 - x_3 + x_4 = -5 \end{cases}$

   (a) Reduce the augmented matrix further to reduced row-echelon form.

   (b) Write down the solution of the original linear system.

15. (10 points) Consider the set $\mathbb{R}^2$, with the addition defined by $[x, y] \oplus [a, b] = [ax, by]$, and with scalar multiplication defined by $r[x, y] = [rx, ry + 1]$. Is this set a vector space?

   If so, $\vec{0} =$ ________________, $-[x, y] =$ ________________.