應數一線性代數 **2020** 春**,** 第一次期中考

學號: __________________________________, 姓名: __________________________________

本次考試共有 11 頁 (包含封面)，有 10 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知**:**

- 請在第一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- 不可翻閱課本或筆記。
- 第一部份是必答題，請務必回答每一題。計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。
- 第二部份是選答題，請在其中挑兩題作答。

高師大校訓：誠敬弘遠

誠，一生動念都是誠實端正的。敬，就是對知識的認真尊重。宏，開拓視界，恢宏心胸。遠，任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: __________________________________

第一部份，必答題，請每一題都要做答

1. (10 points) Let

$$A = \begin{bmatrix} -1 & 0 & 0 \\ -4 & 2 & -1 \\ 4 & 0 & 3 \end{bmatrix}$$

Find (if exists) an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$. Also, find the eigenvalues of A^{100}.

(1) The eigenvalue of A^{100} are ________________. (2) Is A diagonalizable?______

If A diagonalizable, C= ________________, D=________________, and A^{100}= _________________(不需要化簡).

2. (15 points) Find the formula for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects in the line $3x + y = 0$.

Answer: $T([x, y]) =$ ________________

3. (15 points) (a) Solve the system $\begin{cases} x_1' = 2x_1 + 2x_2 \\ x_2' = x_1 + 3x_2 \end{cases}$

(b) Find the solution that satisfies the initial condition $x_1(0) = 2, x_2(0) = 5$.

4. (15 points) Let the sequence $a_0, a_1, ...$ given by $a_0 = 0, a_1 = 1$, and $a_k = a_{k-1} + \frac{3}{4}a_{k-2}$ for $k \geq 2$.
(1) Find the matrix A that can be used to generate this sequence. (2) Estimate(估計) a_k for large k.

Answer: A=________________, a_k=________________.

5. (10 points) Find the projection of [1, 0, 0] on the subspace $W = \mathrm{sp}([2, 1, 1], [1, 0, 2])$ in $\mathbb{R}^3$

   Answer: ________________

6. (15 points) Use Gram-Schmidt process to find an orthonormal basis for the subspace W of $\mathbb{R}^4$ spanned by [1, 0, 1, 0], [1, 1, 1, 0], [1, 0, 1, 1]. Find the QR-factorization of A, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Answer:

Q=________________, R=________________, an orthonormal basis = ________________________

第二部份，選答題，請勾選兩題 予以評分

☐ 本題我要作答

7. (10 points) Let W be a subspace of $\mathbb{R}^n$ and let $\vec{b}$ be a vector in $\mathbb{R}^n$. Prove that there is one and only one vector $\vec{p}$ in W such that $\vec{b} - \vec{p}$ is perpendicular(垂直) to every vector in W.

☐ 本題我要作答

8. (10 points) The **trace** of an $n \times n$ matrix A is defined by

$$tr(A) = a_{11} + a_{22} + ... + a_{nn}.$$

Let the characteristic polynomial $p(A)$ factor(因式分解) into linear factors(一次因式), so that A has n (not necessarily(必須) distinct(不同)) eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$. Prove that

$$\begin{aligned} tr(A) &= (-1)^{n-1}(\text{Coefficient(係數) of } \lambda^{n-1} \text{ in } p(\lambda) \\ &= \lambda_1 + \lambda_2 + ... + \lambda_n \end{aligned}$$

☐ 本題我要作答

9. (10 points) Prove that, for every square(正方形) matrix A all of whose eigenvalues are real, the product of its eigenvalues is $\det(A)$

☐ 本題我要作答

10. (10 points) Prove that, if a matrix is diagonalizable(可對角線化), so is its transpose(轉置).

學號: ______________, 姓名: ______________, 以下由閱卷人員填寫

Question:	1	2	3	4	5	6	Total
Points:	10	15	15	15	10	15	80
Score:							

7	8	9	10	Total
10	10	10	10	20