應數一線性代數 2020 春, 期末考

學號: ______________________________, 姓名: ______________________________

本次考試共有 10 頁 (包含封面)，有 14 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁以及最後一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬弘遠**

誠，一生動念都是誠實端正的。**敬**，就是對知識的認真尊重。**宏**，開拓視界，恢宏心胸。**遠**，任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ______________________________

1. (10 points) Find the projection matrix P for the plane $W : 3x + 2y + z = 0$ in $\mathbb{R}^3$ and the find the projection $\vec{b}_w$ of $\vec{b} = [4, 2, -1]$ on it.

   Answer: $\vec{b}_w =$ ________________, $P =$ ________________.

2. (10 points) Find the lease squares straight line fit to the four points (0,1) (1,3) (2,4) (3,4) and use it to approximate the fifth points (4, a).

   Answer: the line equation =________________, a=________________.

3. (5 points) Find the coordinate vector of $\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ in M_2 relative to $\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right)$

Answer: ____________________

4. (10 points) Find the five fifth roots of $-243i$. (need not simplify)

5. (10 points) Find the matrix representations $R_{B,B}$, $R_{B',B'}$ and an invertible C such that $R_{B',B'} = C^{-1}R_{B,B}C$ for the given linear transformation T.
$T : \mathbb{R}^3 \to \mathbb{R}^3$ defined as <u>reflection</u> of $\mathbb{R}^3$ through the plane $x+y-z = 0$; $B = E$, $B' = ([1, 0, 1], [1, -1, 0], [1, 1, -1])$.

$C_{BB'}$=________________, $C_{BB'}$=________________, $R_{B'B'}$=________________ and R_{BB}=________________.
Is C=$C_{BB'}$ or $C_{BB'}$? __________.

6. (5 points) Express $(\sqrt{3}+i)^6$ in the form $a+bi$ for a, b are real numbers.

   Answer: a=__________, b=__________.

7. (10 points) Using the Gram-Schmidt process to transform the basis $\{[1, i, 1-i], [1+i, 1-i, 1]\}$ into an orthogonal basis and then extend it as an orthogonal basis for $\mathbb{C}^3$.

   Answer: the found orthogonal basis for $\mathbb{C}^3$ is ______________________________

8. (10 points) Find an unitary matrix U and a diagonal matrix D such that $D = U^{-1}AU$. Also find where

$$A = \begin{bmatrix} 1 & -i & 0 \\ i & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

9. (10 points) Find a Jordan canonical form and a Jordan basis for the given matrix.

$$\begin{bmatrix} i & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 2 & 0 & -1 & 0 & 2 \end{bmatrix}$$

10. (10 points) Find a polynomial in A that gives the zero matrix.

$$\begin{bmatrix} i & 0 & 0 & 0 & 0 \\ 0 & i & 1 & 0 & 0 \\ 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

11. (5 points) Prove or disprove the following: All 2×2 matrix with determinant 1 is an orthogonal matrix.

12. (10 points) Find all the possible 2×2 real matrix that is unitarily diagonalizable.

13. (5 points) Prove that for $\vec{u}, \vec{v} \in \mathbb{C}^n$, $(\vec{u}^* \vec{v})^* = \overline{\vec{u}^* \vec{v}} = \vec{v}^* \vec{u} = \vec{u}^T \overline{\vec{v}}$

14. (10 points) Prove the following:
    (a) Show that every Hermitian matrix is normal.
    (b) Show that every unitary matrix is normal.
    (c) Show that, if $A^* = -A$, then A is normal.

應數一線性代數期末考, 學號: ______________, 姓名: ______________, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
Points:	10	10	5	10	10	5	10	10	10	10	5	10	5	10	120
Score:															