應數一線性代數 2020 秋, 第一次期中考

學號: ______________________________, 姓名: ______________________________

本次考試共有 8 頁 (包含封面)，有 13 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬弘遠**

誠，一生動念都是誠實端正的。**敬**，就是對知識的認真尊重。**宏**，開拓視界，恢宏心胸。**遠**，任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ______________________________

1. (5 points) Find the value of x such that $[x, -3, 5]$ is perpendicular to $[-1, 3, 4]$

   Answer: x=________________________________

2. (10 points) Solve the given linear system and express the solution set.

$$\begin{cases} x_1 - x_2 + x_3 + x_4 = 5 \\ \phantom{x_1 -{}} x_2 - x_3 + 2x_4 = 8 \\ x_1 \phantom{{}- x_2} - 4x_3 + 3x_4 = 13 \end{cases}$$

   Answer: the solution set is

   ____

3. (10 points) Assume the the matrix A can be row reduces to H, please answer the following questions.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 3 \\ 1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(a) the **rank** of matrix A, is ______________.

(b) a basis for the **row space** of A is ____.

(c) a basis for the **column space** of A is .

____

(d) a basis for the **nullspace** of A is .

____

4. (5 points) If a 8×11 matrix A has rank 5, find the dimension of the column space of A, the dimension of the nullspace of A, and the dimension of the row space of A.

5. (10 points) Given set $S = \{[-2, 2, 3, 0], [1, -2, 1, 0], [-1, 0, 4, 0]\}$ in $\mathbb{R}^4$.

   (a) Determine whether the set S is linearly dependent or linearly independent. If it is linearly dependent, find a basis for $sp(S)$.

   Answer: ____.

   (b) Enlarge the basis you found in part (a) to be a basis for $\mathbb{R}^4$.

6. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T([1,0,0]) = [1,2,1], T([0,1,0]) = [3,0,4]$, and $T([1,0,1]) = [5,4,6]$.

   (a) Find the standard matrix representation of T.

   (b) Use the standard matrix representation to find a formula for $T([x_1, x_2, x_3])$.

   (c) Find the kernel of T.

   (d) Is the linear transformation T invertible? If so, find the standard matrix representation of T^{-1}.

7. (10 points) (a) Compute the inverse of A and verify that you have the correct inverse.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ 3 & 1 & 0 \end{bmatrix} \qquad \text{Answer: } A^{-1} = \underline{\hspace{6em}}.$$

(b) Use part (a) to solve

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

8. (5 points) Determine if the set $W = \{(x, y, z) \in \mathbb{R}^3 | z = 3x + 2\}$ is a subspace of $\mathbb{R}^3$

9. (5 points) Circle True or False. Read each statement in original Greek before answering.
   (a) True False If A is a 2×3 matrix and B is a 2×4 matrix, then AB is a 3×4 matrix.
   (b) True False Any six vectors in $\mathbb{R}^4$ must span $\mathbb{R}^4$.
   (c) True False If T is a linear transformation, then $T(0) = 0$.
   (d) True False No vector is its own additive inverse.
   (e) True False If $\{v_1, v_2, ..., v_n\}$ generates V, then each $v \in V$ is a unique linear combination of vectors in this set.

10. (5 points) Let F bet he set of all real-valued functions on a (nonempty) set S; that is , let F be the set of all functions mapping S into $\mathbb{R}$. For $f, g \in F$, let the sum $f \oplus g$ of two functions f and g in F, and for any scalar r, let scalar multiplication be defined below. Is this set a vector space?

$$(f \oplus g)(x) = \max\{f(x), g(x)\} \quad \text{for all } x \in S$$
$$(rf)(x) = rf(x) \quad \text{for all } x \in S$$

11. (5 points) The set of all functions f such that $f(0) = 1$ in the vector space F of all functions mapping $\mathbb{R}$ into $\mathbb{R}$.

12. (10 points) Let W_1 and W_2 be two subspace of $\mathbb{R}^n$. Prove that their intersection $W_1 \cap W_2$ is also a subspace.

13. (10 points) Prove that the given relation holds for all vectors, matrices and scalars for which the expression are defined.

$$(AB)^T = B^T A^T$$

學號: ______________, 姓名: ______________, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Points:	5	10	10	5	10	10	10	5	5	5	5	10	10	100
Score:														