應數一線性代數 2021 春, 期中考試卷 A

學號: ________________________, 姓名: ________________________

本次考試共有 9 頁 (包含封面)，有 9 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- 不可翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。
沒有計算過程，就算回答正確答案也不會得到滿分。
答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：誠敬宏遠

誠，一生動念都是誠實端正的。　敬，就是對知識的認真尊重。
宏，開拓視界，恢宏心胸。　遠，任重致遠，不畏艱難。

請尊重自己也尊重其他同學，考試時請勿東張西望交頭接耳。

1. (10 points) Let

$$A = \begin{bmatrix} -1 & 0 & 1 \\ -7 & 2 & 5 \\ 3 & 0 & 1 \end{bmatrix}$$

Find (if exists) an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$. Also, find the eigenvalues of A^{100}.

(1) Is A diagonalizable?______. If A diagonalizable, C= ___________, D=___________.

(2) The eigenvalue of A are ________________. The eigenvalue of A^{100} are ________________.

2. (15 points) Find the formula for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects in the line $x + 5y = 0$.

   Answer: $T([x, y]) =$ ________________

3. (15 points) (a) Solve the system $\begin{cases} x_1' = 3x_1 + 2x_2 \\ x_2' = x_1 + 2x_2 \end{cases}$

   (b) Find the solution that satisfies the initial condition $x_1(0) = 2, x_2(0) = 5$.

   Answer: ______________________________________

4. (10 points) Find the projection matrix P for the plane $W : 2x + 2y + z = 0$ and then find the projection of $\vec{b} = [4, 2, -1]$ on the plane.

   Answer: $\vec{b}_W =$ ____________________, $P =$ ____________________.

5. (10 points) Find the least-square solution of the below system.

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ -2 \end{bmatrix}$$

Answer: The least-square solution = ________________.

6. (15 points) Use Gram-Schmidt process to find an orthonormal basis for the subspace W of $\mathbb{R}^4$ spanned by [1, 1, 0, 0], [1, 1, -1, 0], [1, 0, 1, 1] and then use it to find the QR-factorization of A, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Answer : Q=_____________, R=_____________, an orthonormal basis = ________________

7. (10 points) Let W be a subspace of $\mathbb{R}^n$ and let $\vec{b}$ be a vector in $\mathbb{R}^n$. Prove that there is one and only one vector $\vec{p}$ in W such that $\vec{b} - \vec{p}$ is perpendicular(垂直) to every vector in W.

8. (10 points) Let A and C be orthogonal $n \times n$ matrices. Show that CAC^{-1} is orthogonal.

9. (5 points) Circle True or False. Read each statement in original Greek before answering.

(a) True False If $\vec{v}$ is an eigenvector of an invertible matrix A, then $c\vec{v}$ is an eigenvector of A^{-1} for all nonzero scalar c.

(b) True False Every $n \times n$ real symmetric matrix is real diagonalizable.

(c) True False The intersection of W and $W^{\perp}$ is empty.

(d) True False A square matrix is orthogonal if its column vectors are orthogonal.

(e) True False The least-square solution vector of $A\vec{x} = \vec{b}$ is the projection of $\vec{b}$ on the column space of A.

學號: ____________, 姓名: ____________, 以下由閱卷人員填寫

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	15	15	10	10	15	10	10	5	100
Score:										