應數一線性代數 2021 秋, 第一次期中考

學號: ______________________________, 姓名: ______________________________

本次考試共有 8 頁 (包含封面)，有 12 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁及最後一頁填上姓名學號。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。
- 書寫空間不夠時，可利用試卷背面，但須標記清楚。

高師大校訓：**誠敬宏遠**

誠: 一生動念都是誠實端正的。 **敬**: 就是對知識的認真尊重。
宏: 開拓視界，恢宏心胸。 **遠**: 任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ______________________________

1. (5 points) Find all numbers r such that $\begin{bmatrix} 2 & 4 & 2 \\ 1 & r & 3 \\ 1 & 1 & 2 \end{bmatrix}$ is invertible.

   Answer: r=____

2. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T([1,0,0]) = [2,3,0], T([0,1,0]) = [-2,0,1]$, and $T([1,2,3]) = [4,15,2]$. Find $T^{-1}([4,-3,2]) =$ ________________________________

3. (5 points) Classify $\vec{v} = [4, 1, 2, 1, 6]$ and $\vec{u} = [8, 2, 4, 2, 3]$ are parallel, perpendicular, or neither.

   Answer: $\vec{v}$ and $\vec{u}$ are ______________________________ .

4. (10 points) Find the homogeneous solution and general solution of the given linear system and express the solution set.

$$\begin{cases} x_1 \qquad + \ x_3 + 5x_4 = -1 \\ \qquad x_2 + 2x_3 + 6x_4 = 3 \\ x_1 - x_2 \qquad + 2x_4 = 3 \end{cases}$$

   Answer: the homogeneous solution is ______________________________.

   The general solution is ______________________________.

5. (10 points) Assume the the matrix A can be row reduces to H, please answer the following questions.

$$A = \begin{bmatrix} 5 & 1 & 0 & 3 & -3 \\ 1 & 0 & -1 & 1 & 8 \\ 0 & 3 & 1 & -6 & 1 \\ 1 & 1 & 0 & -1 & 7 \end{bmatrix}, H = \begin{bmatrix} 5 & 0 & 0 & 5 & 0 \\ 0 & 2 & 0 & -4 & -6 \\ 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) the **rank** of matrix A, is ____________________.

(b) a basis for the **row space** of A is ____________________.

(c) a basis for the **column space** of A is ____________________.

(d) a basis for the **nullspace** of A is ____________________.

6. (5 points) Suppose that T is a linear transformation with standard matrix representation A, and that A is a 8×11 matrix such that the nullspace of A has dimension 6. What is the dimension of the range of T?

7. (10 points) Given set $S = \{[-2, 3, 1, 0], [0, 1, 5, -2], [1, -1, 2, -1]\}$ in $\mathbb{R}^4$.

   (a) Determine whether the set S is linearly dependent or linearly independent. If it is linearly dependent, find a basis for $sp(S)$.

   Answer: ____.

   (b) Enlarge the basis you found in part (a) to be a basis for $\mathbb{R}^4$.

   Answer: ____

8. (10 points) Determine if the set $W = \{(x, y, z) \in \mathbb{R}^3 | x = 2y + z, y = 5z\}$ is a subspace of $\mathbb{R}^3$

9. (5 points) Let F bet he set of all real-valued functions on a (nonempty) set S; that is , let F be the set of all functions mapping S into $\mathbb{R}$. For $f, g \in F$, let the sum $f \oplus g$ of two functions f and g in F, and for any scalar r, let scalar multiplication be defined below. Is this set a vector space?

$$(f \oplus g)(x) = f(x) + 3g(x) \quad \text{for all } x \in S$$
$$(rf)(x) = rf(x) \quad \text{for all } x \in S$$

10. (10 points) Let $\vec{v}_1$ and $\vec{v}_2$ be two vectors in $\mathbb{R}^n$. Prove that $sp(\vec{v}_1, \vec{v}_1 + \vec{v}_2) = sp(\vec{v}_1 - \vec{v}_2, \vec{v}_1 + \vec{v}_2)$

11. (10 points) Suppose that the vectors $\vec{v}, \vec{w}$, and $\vec{x}$ are mutually perpendicular (i.e. $\vec{v}$ and $\vec{w}$ are perpendicular, $\vec{v}$ and $\vec{x}$ are perpendicular, and $\vec{w}$ and $\vec{x}$ are perpendicular). Use dot products to find $\|\vec{v} + 3\vec{w} + 2\vec{x}\|$ in terms of the magnitudes (lengths) of $\vec{v}, \vec{w}$, and $\vec{x}$. Hint: Start by computing $\|\vec{v} + 3\vec{w} + 2\vec{x}\|^2$.

12. (10 points) Let A and C be matrices such that the product AC is defined. Whether the column space of AC is contained in the column space of A or C? Explain your answer.

    Answer: the column space of AC is contained in the column space of ________________.

學號: ____________, 姓名: ____________, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	5	10	5	10	10	5	10	10	5	10	10	10	100
Score:													