應數一線性代數 2021 秋, 期末考

學號: ______________________________, 姓名: ______________________________

本次考試共有 7 頁 (包含封面)，有 11 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁及最後一頁填上姓名學號。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。
- 書寫空間不夠時，可利用試卷背面，但須標記清楚。

高師大校訓：**誠敬宏遠**

誠: 一生動念都是誠實端正的。 **敬**: 就是對知識的認真尊重。
宏: 開拓視界，恢宏心胸。 **遠**: 任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ______________________________

1. (5 points) Find $\mathbf{v}_B$, which is the coordinate vector of the given vector $\mathbf{v} = [1, 2, -2]$ relative to the indicated ordered basis $\mathbf{B} = \{[1, 1, 1], [1, 2, 0], [1, 0, 1]\}$.

2. (10 points) Find the coordinate vector of polynomial $4x^3 + 5x^2 + x$ relative to the ordered basis $B = ((x+1)^3, (x+1)^2, (x+1), 1)$ of the vector space P_3 of polynomials of degree at most 3.

3. (10 points) Let $V = sp(e^{2x}, e^{4x}, e^{8x})$, $V' = sp(e^{3x}, e^{7x}, e^{9x})$ are the subspaces of the vector space of all real-valued functions with domain $\mathbb{R}$, and let $B = (e^{2x}, e^{4x}, e^{8x})$, $B' = (e^{3x}, e^{7x}, e^{9x})$. Let $T : V \longrightarrow V'$ be the linear transformation having the given matrix A as matrix representation relative to B, B'.

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 3 & 1 \\ -2 & 1 & 3 \end{bmatrix},$$

Find $T(ae^{2x} + be^{4x} + ce^{8x}) =$____________________

4. (10 points) Suppose that A is a 4×4 matrix with determinant 3.

(a) Find $\det(5A) =$ _______________

(b) Find $\det(A^{-1}) =$ _______________

(c) Find $\det(2A^{-1}) =$ _______________

(d) Find $\det((2A)^{-1}) =$ _______________

5. (10 points) Suppose that A is a 3×3 matrix with row vectors $\vec{a}, \vec{b}$, and $\vec{c}$, and that $det(A) = 3$. Find the determinant of the matrix having $2\vec{a} + 3\vec{b} + 2\vec{c}, \vec{c}, \vec{a}$ as its row vectors

Determinant = ________________

6. (10 points)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$$

The inverse of A = ________________ , and the adjoint matrix of A = ________________

7. (5 points) Let $\vec{a} = \vec{i} - 2\vec{k}$, $\vec{b} = -\vec{i} + 3\vec{j}$, $\vec{c} = \vec{i} + 2\vec{j} + \vec{k}$.

   Find $\vec{a} \cdot (\vec{b} \times \vec{c}) =$ ________________

8. (10 points) Find out whether points (0, 0, 0, 0) (2, 1, 0, 0), (3, -2, 0, 0), (0, 0, 2, 6) and (0, 0, 2, 3) lie in a plane in $\mathbb{R}^4$.

   Answer: ________________

9. (10 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T([x, y]) = [y, x, 3x + y]$. Let D be the rectangle $2 \leq x \leq 3$, $-1 \leq y \leq 4$. FInd the area of $T(D)$.

   Answer: ________________

10. (10 points) Circle True or False. **Write down the explanation if the statement is False.** Read each statement in original Greek before answering.

    (a) True False There's an unique coordinate vector associated with each vector $\vec{v}inV$ relatve to an ordered basis for V

    (b) True False For every vector $\vec{b}'$ in V', the function$T_{\vec{b}'} : V \to V'$ defined by $T_{\vec{b}'}(\vec{v}) = \vec{b}'$ for all $\vec{v}$ in V is a linear transformation.

    (c) True False The parallelogram (平行四邊形) in $\mathbb{R}^2$ determined by non-zero vectors $\vec{a}, \vec{b}$ is a square (正方形) if and only if $\vec{a} \cdot \vec{b} = 0$

    (d) True False The product of a square matrix and its adjoint is the identity matrix.

    (e) True False There is no square matrix A such that $\det(A^T A) = -1$.

11. (10 points) Let $S = \{1, \sin(x), \cos(x), \sin(2x), \cos(2x)\}$ is a set of functions in the vector space F of all functions mapping $\mathbb{R}$ into $\mathbb{R}$.

    (a) Prove that S is an independent set in F.

    (b) Find a basis for the subspace of F generated by the functions $\{f_1, f_2, f_3, f_4\}$, where

$$f_1(x) = 1 + 2\sin(x) + \cos(2x) \quad f_2(x) = -2 + 4\sin(x) - \cos(x) + \cos(2x)$$

$$f_3(x) = 7 - 2\sin(x) + 2\cos(x) + \cos(2x) \quad f_4(x) = 4 + \sin(x) + \cos(x) + 2\sin(2x) - \cos(2x)$$

學號: ______________, 姓名: ______________, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	5	10	10	10	10	10	5	10	10	10	10	100
Score:												