應數一線性代數 2022 春, 期中考

學號: _____, 姓名: _____

本次考試共有 11 頁 (包含封面)，有 10 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。
  沒有計算過程，就算回答正確答案也不會得到滿分。
  答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬宏遠**

誠，一生動念都是誠實端正的。 **敬**，就是對知識的認真尊重。
宏，開拓視界，恢宏心胸。 **遠**，任重致遠，不畏艱難。

請尊重自己也尊重其他同學，考試時請勿東張西望交頭接耳。

1. (10 points) (a) Solve the system $\begin{cases} x_1' = 3x_1 - 5x_2 \\ x_2' = 2x_2 \end{cases}$

   (b) Find the solution that satisfies the initial condition $x_1(0) = 2, x_2(0) = 5$.

   Answer: _____

2. (10 points) Let

$$A = \begin{bmatrix} 9 & -3 & 3 \\ -2 & 10 & 2 \\ 1 & 1 & 11 \end{bmatrix}$$

Find (if exists) an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$. Also, find the eigenvalues of A^{100}.

(1) Is A diagonalizable?_____. If A diagonalizable, C= _____, D=_____.

(2) The eigenvalue of A are _____. The eigenvalue of A^{100} are _____.

3. (10 points) Find the formula for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects in the line $3x + 2y = 0$.

   Answer: $T([x, y]) =$ _____

4. (10 points) Find all the possible a, b, c, d, x, y so that the matrix A is orthogonal.

$$A = \begin{bmatrix} a & y & 0 \\ 2x & 3y & c \\ x & b & d \end{bmatrix}$$

5. (10 points) Find the projection matrix P for the plane $W : 2x - y + 2z = 0$ and then find the projection of $\vec{b} = [3, 2, 1]$ on the plane.

   Answer: $\vec{b}_W =$ _____, $P =$ _____.

6. (10 points) Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

Factor A in the form $A = QR$, where Q is an orthogonal matrix and R is an upper-triangular invertible matrix.

Answer

Q=_____, R=_____,

7. (10 points) Find the least squares straight line fit to the five points (-4, -2), (-2, 0), (0,1), (2, 4), (4, 5) and use it to approximate the fifth points (1, a).

   Answer: the line equation =_____, a=_____.

8. (10 points) Prove that, for every square matrix A all of whose eigenvalues are real, the product of its eigenvalues is $\det(A)$

9. (10 points) Show that the real eigenvalue of an orthogonal matrix must be equal to 1 or -1.

   Hint: Think in terms of linear transformations.

10. (10 points) Circle True or False and disprove the statement if it is FALSE. Read each statement in original Greek before answering.

(a) True False A square matrix is orthogonal if its column vectors are orthogonal.

(b) True False Every invertible matrix is diagonalizable.

(c) True False The intersection of W and $W^{\perp}$ is empty.

(d) True False The least-square solution vector of $A\vec{x} = \vec{b}$ is the projection of $\vec{b}$ on the column space of A.

(e) True False If λ is an eigenvalue of a matrix A, then λ is an eigenvalue of $A + cI$ for all nonzero scalar c.

學號: _____, 姓名: _____, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	10	10	10	10	10	10	10	100
Score:											