應數一線性代數 2022 秋, 第一次期中考

學號: _____, 姓名: _____

本次考試共有 8 頁 (包含封面)，有 12 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁及最後一頁填上姓名學號。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。
- 書寫空間不夠時，可利用試卷背面，但須標記清楚。

高師大校訓：**誠敬宏遠**

誠: 一生動念都是誠實端正的。 **敬**: 就是對知識的認真尊重。
宏: 開拓視界，恢宏心胸。 **遠**: 任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: _____

1. (10 points) (a) Find the inverse of the matrix A, if it exists, and (b) express the inverse matrix as a product of elementary matrices. $A = \begin{bmatrix} 5 & 2 \\ 3 & 8 \end{bmatrix}$

   Answer: (a) _____, (b) _____

2. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T([1, 0, 0]) = [2, 4, 0], T([0, 1, 0]) = [1, 0, 3]$, and $T([2, 1, 3]) = [11, 23, 3]$. Find $T([4, -3, 2]) =$ _____

3. (5 points) Find all possible scalar c such that the vector $\vec{i} + c\vec{j} - 3\vec{k}$ is in the span of $\vec{i} + \vec{j} - \vec{k}$ and $\vec{j} + 3\vec{k}$.

   Answer: $c =$ _____.

4. (5 points) Given two vectors $\vec{v} = [4, x, 2, 1]$ and $\vec{u} = [8, 2, 4, y]$. Find all $x, y \in \mathbb{R}$ so that

   (a) $\vec{v}, \vec{u}$ are parallel. _____.

   (b) $\vec{v}, \vec{u}$ are perpendicular. _____.

5. (5 points) Suppose that T is a linear transformation with standard matrix representation A, and that A is a 9×15 matrix such that the nullspace of A has dimension 5.

   (a) What is the dimension of the range of T?_____.

   (b) What is the dimension of the kernel of T?_____.

6. (10 points) Consider the given linear system:

$$\begin{cases} x_1 - x_2 \quad + \quad x_4 = 1 \\ x_1 \quad - x_3 + 2x_4 = 0 \\ \quad - x_2 + x_3 + \quad x_4 = -6 \end{cases}$$

(a) Write its associated augmented matrix. _____

(b) Reduce the matrix to its reduced row-echelon form (rref). _____.

(c) Find the homogeneous solution of the linear system . _____.

(d) Find the general solution of the linear system . _____.

7. (10 points) Determine if the line $y = mx$ is a subspace of $\mathbb{R}^2$. *Hint:* Write the line as a set $W = \{[x, mx] | x \in \mathbb{R}\}$.

8. (10 points) Assume the the matrix A can be row reduces to H, please answer the following questions.

$$A = \begin{bmatrix} 5 & 3 & 1 & 2 & 19 & 5 \\ 1 & 1 & 1 & 0 & 3 & -1 \\ 0 & 2 & 4 & -1 & -4 & -9 \\ 1 & -1 & -3 & -4 & 7 & 3 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 & -1 & 0 & 5 & 3 \\ 0 & 1 & 2 & 0 & -2 & -4 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) the **rank** of matrix A, is _____.

(b) a basis for the **row space** of A is _____.

(c) a basis for the **column space** of A is _____.

(d) a basis for the **nullspace** of A is _____.

9. (5 points) Consider the set $\mathbb{R}^2$, with the addition defined by $[x, y] \oplus [a, b] = [x + a + 1, y + b]$, and with scalar multiplication defined by $r \otimes [x, y] = [rx - 1, ry]$.

   a. Is this set a vector space? _____
   Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. What is the zero vector in this vector space? *Hint:* The zero vector may NOT be the vector $[0, 0]$.
   Answer: the zero vector is _____, for any vectors [x,y], the -[x,y] is _____

10. (10 points) For vectors $\vec{u}, \vec{v}$ and $\vec{w}$ in $\mathbb{R}^n$ and for scalars r and s, prove that, if $\vec{w}$ is perpendicular to both $\vec{v}$ and $\vec{u}$, then $\vec{w}$ is perpendicular to $r\vec{u} + s\vec{v}$.

11. (10 points) Prove that the given relation holds for all matrices for which the expressions are defined.

$$(AB)C = A(BC)$$

12. (10 points) Circle each of the following True or False and then give a counterexample （反例）for the false statement.

    1. True False For all positive integers m and n, the nullity of an $m \times n$ matrix might be any number from 0 to n.

    2. True False There are exactly two unit vectors perpendicular (垂直) to any given nonzero vectors in $\mathbb{R}^n$.

    3. True False Every independent subset of $\mathbb{R}^n$ is a subsets of every basis for $\mathbb{R}^n$.

    4. True False Every vector spaces has at least two vectors.

    5. True False Every function mapping $\mathbb{R}^n$ into $\mathbb{R}^m$ is a linear transformation.

學號: _____, 姓名: _____, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	10	10	5	5	5	10	10	10	5	10	10	10	100
Score:													