應數一線性代數 2022 秋, 期末考

學號: _____, 姓名: _____

本次考試共有 8 頁 (包含封面)，有 10 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁及最後一頁填上姓名學號。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。
- 書寫空間不夠時，可利用試卷背面，但須標記清楚。

高師大校訓：**誠敬宏遠**

誠: 一生動念都是誠實端正的。 **敬**: 就是對知識的認真尊重。
宏: 開拓視界，恢宏心胸。 **遠**: 任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: _____

1. (10 points) (a) Determine the set S_1 of all functions f such that $f(0) = 1$ is a subspace in the vector space F of all functions mapping $\mathbb{R}$ into $\mathbb{R}$.

   Answer: Is S_1 a subspace of F? _____

   (b) Determine the set of all functions f such that $f(1) = 0$ is a subspace in the vector space F of all functions mapping $\mathbb{R}$ into $\mathbb{R}$.

   Answer: Is S_2 a subspace of F? _____

2. (10 points) Find the area of the parallelogram(平行四邊形) in $\mathbb{R}^3$ determined by the vectors $[2, -3, 5]$ and $[4, -5, 1]$

   Answer: area = _____

3. (10 points) Find the coordinate vector of the given vector relative to the indicated ordered basis.

   $7x^3 + 3x^2 - 2x + 3$ in P_3 relative to $(x^2 + x, x^3 + 2x - 1, x^3 + x, 2x^2 + 1)$

   Answer: the coordinate vector is _____

4. (10 points) Let $T : P_2 \to P_3$ be defined by $T(p(x)) = (x-2)p(x+1)$, the ordered basis for P_2 is $B = (x^2, x, 1)$ and the ordered basis for P_3 is $B' = (x^3, x^2, x, 1)$. Fine the standard matrix representation A of T relative to the ordered bases B and B'.

Answer: (a) $A =$ _____

(b) $T(-2x^2 - 4x + 3) =$ _____

(c) The $ker(T) =$ _____

5. (10 points) Let V and V' be vector spaces with ordered bases $B = ([1,3,-2],[4,1,2],[-1,1,0])$ and $B' = ([1,0,1,0],[2,1,1,-1],[0,1,1,-1],[2,0,3,1])$, respectively, and let $T : V \longrightarrow V'$ be the linear transformation having the given matrix A as matrix representation relative to B, B'. For a vector $\vec{v}$ such that $\vec{v}_B = [1,-3,10]$, find $T(\vec{v})$.

$$A = \begin{bmatrix} 0 & 4 & -1 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix},$$

Answer: If $\vec{v}_B = [1,-3,10]$, then $\vec{v} =$ _____.
$T(\vec{v}) =$ _____.

6. (10 points) Suppose that A is a 5×5 matrix with determinant 2.

   (a) Find $\det(3A) =$ _____.

   (b) Find $\det(A^{-1}) =$ _____.

   (c) Find $\det(7A^{-1}) =$ _____.

   (d) Find $\det((7A)^T) =$ _____.

7. (10 points)

$$A = \begin{bmatrix} 0 & 3 & 1 \\ 5 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$$

The inverse of A = _____ , and the adjoint matrix of A = _____

8. (10 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T([x, y]) = [y, x, x + y]$. Find the volume of the image under T of the disk $x^2 + y^2 \leq 16$.

   Answer: _____.

9. (10 points) Show that $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$ for any vectors $\vec{a}, \vec{b}$ and $\vec{c}$ in $\mathbb{R}^3$.

10. (10 points) Circle each of the following True or False. Please give a counterexample (反例) for the false statement and give an explain (解釋) for the true statement.

(a) True False A linear transformation $T : V \to V'$ carries a pair $\vec{v}$, $-\vec{v}$ in V into a pair $\vec{v}'$, $-\vec{v}'$ in V'

(b) True False The product of a square matrix and its adjoint is the identity matrix.

(c) True False There is no square matrix A such that $\det(A^T A) = -1$.

(d) True False The determinant of 3×3 matrix is zero if the points in $\mathbb{R}^3$ given by the rows of the matrix lie in a plane.

(e) True False Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. The image under T of an n-box in $\mathbb{R}^n$ of. volume > 0 is a n-box in $\mathbb{R}^n$ of volume > 0

學號: _____, 姓名: _____, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	10	10	10	10	10	10	10	100
Score:											