應數一線性代數 2023 秋, 期末考

學號: ______________________, 姓名: ______________________

本次考試共有 10 頁 (包含封面)，有 11 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁及最後一頁填上姓名學號。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。
- 書寫空間不夠時，可利用試卷背面，但須標記清楚。

高師大校訓：**誠敬宏遠**

誠: 一生動念都是誠實端正的。 **敬**: 就是對知識的認真尊重。
宏: 開拓視界，恢宏心胸。 **遠**: 任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ______________________

1. (10 points) Find the coordinate vector of the given vector relative to the indicated ordered basis.
$\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ in M_2 relative to $\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right)$.

   Answer: the coordinate vector is ____________________

2. (10 points) Let $\vec{a} = \vec{i} - 3\vec{k}$, $\vec{b} = -\vec{i} + 4\vec{j}$, $\vec{c} = \vec{i} + 2\vec{j} + \vec{k}$.
   Find $\vec{a} \cdot (\vec{b} \times \vec{c}) =$ ____________________

3. (10 points) Let $T : P_2 \to P_3$ be defined by $T(ax^2+bx+c) = (4a+b-c)x^3+(2a+2b)x^2+(6b+c)x+(2a+b+3c)$, the ordered basis for P_2 is $B = (x^2, x, 1)$ and the ordered basis for P_3 is $B' = (x^3, x^2, x, 1)$. Fine the matrix representation A of T relative to the ordered bases B and B'.

Answer: (a) $A =$ ____________________

(b) find $p(x)$ such that $T(p(x)) = 7x^3 + 4x^2 + 4x - 3$. $p(x) =$ ____________________

4. (10 points) Find the determinant of the given matrix.

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 2 & 4 \\ 2 & 3 & 1 & 4 & 2 & 4 \\ 4 & 6 & 0 & 8 & 2 & 4 \\ -1 & 1 & 0 & -1 & 3 & -5 \\ 0 & 0 & 0 & 0 & 5 & 7 \\ 1 & 2 & 0 & -1 & 2 & 5 \end{bmatrix}$$

Answer: $\det(A) =$ ____________________

5. (10 points)

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

The inverse of A = ________________, and the adjoint matrix of A = ________________

6. (10 points) Determine the set S_1 of all functions f such that $f(0) = 1$ is a subspace in the vector space F of all functions mapping $\mathbb{R}$ into $\mathbb{R}$.

Answer: Is S_1 a subspace of F? ________________

7. (10 points) Determinant whether the given 4 points lie in a plane in $\mathbb{R}^4$. If so, find its area. If not, find its volume.

$$A(2,1,1,1), B(3,1,-1,2), C(2,0,2,3), D(2,-1,2,0)$$

Answer:

☐ $ABCD$ are coplanar(共平面), and the area of the quadrilateral (四邊形) is ____________________.

☐ $ABCD$ are NOT coplanar, and the volume of the tetrahedron(四面體) is ____________________.

8. (10 points) Consider the set $\{(x,y)|\ x+y=0\} \in \mathbb{R}^2$, with the addition defined by $[x,y] \oplus [a,b] = [x+a, y+b]$, and with scalar multiplication defined by $r \otimes [x,y] = [ry, rx]$.

   a. Is this set a vector space? ______________________________

      Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. If it is a vector space, then what is the requested vectors in this vector space?

      Hint: The zero vector may NOT be the vector $[0,0]$.

      Answer: the zero vector is ______________, for any vectors [x,y], the -[x,y] is ______________

9. (10 points) Consider the set $\{(x,y)|\ x+y=1\} \in \mathbb{R}^2$, with the addition defined by $[x,y] \oplus [a,b] = [x+a+1, y+b]$, and with scalar multiplication defined by $r \otimes [x,y] = [rx+r-1, ry]$.

   a. Is this set a vector space? ________________

      Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. If it is a vector space, then what is the requested vectors in this vector space?

      Hint: The zero vector may NOT be the vector $[0,0]$.

      Answer: the zero vector is ____________, for any vectors [x,y], the -[x,y] is ____________

10. (10 points) Determine the dimension of the given set S. Then reduce the given set to be a basis for $sp(S)$.

$$S = sp(1, 4x+5, 5x-4, x^2+2, x-2x^2) \text{ is a subspce in a vector space } P.$$

Answer: $\dim(S) =$ ________________________.

A basis for S is ________________________

11. (10 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ and $\hat{T} : \mathbb{R}^m \to \mathbb{R}^k$ be linear transformations. Prove directly from its definition that $(\hat{T} \circ T) : \mathbb{R}^n \to \mathbb{R}^k$ is also a linear transformation.

學號: ______________, 姓名: ______________, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	10	10	10	10	10	10	10	10	10	10	10	110
Score:												