應數一線性代數 2024 春, 期中考

學號: _____, 姓名: _____

本次考試共有 10 頁 (包含封面)，有 10 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號，忘記填寫扣十分！
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。
  沒有計算過程，就算回答正確答案也不會得到滿分。
  答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬宏遠**

誠，一生動念都是誠實端正的。　**敬**，就是對知識的認真尊重。
宏，開拓視界，恢宏心胸。　**遠**，任重致遠，不畏艱難。

請尊重自己也尊重其他同學，考試時請勿東張西望交頭接耳。

1. (10 points) Let

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

Is A orthogonal diagonalizable? (Yes / No) .

why? _____

2. (10 points) Let

$$A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 0 & -4 \\ 6 & -4 & 3 \end{bmatrix}$$

Is A orthogonal diagonalizable? (Yes / No) .

why? _____

3. (10 points) Solve the system $\begin{cases} x_1' = 4x_1 - 2x_2 + x_3 \\ x_2' = -2x_1 + 3x_2 - 2x_3 \\ x_3' = x_1 - 2x_2 + 4x_3 \end{cases}$

Answer: _____ .

4. (15 points) Use Gram-Schmidt process to find an orthonormal basis for the subspace W of $\mathbb{R}^4$ spanned by [1, 0, 1, 0], [1, 1, -1, 0], [1, 1, 0, 1] and then use it to find the QR-factorization of A, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Answer

Q=_____, R=_____, an orthonormal basis of W = _____

5. (10 points) Find the formula for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects in the line $3x - 7y = 0$.

Answer: $T([x, y]) =$ _____.

6. (10 points) Find the projection of [2, 4, 1] on the plane $P:\ 2x - y - 2z = 0$

Answer: the projection = _____, and the $P^{\perp}$ = _____.

7. (10 points) Show that orthogonal matrices preserve the dot product of vectors. (i.e. $(A\vec{x} \cdot A\vec{y} = \vec{x} \cdot \vec{y})$.)

8. (10 points) Let A is an $n \times n$ invertible matrix and if λ is an eigenvalue of A with $\vec{v}$ as a corresponding eigenvector. Prove that (a) $\lambda \neq 0$ and (b) $1/\lambda$ is an eigenvalue of A^{-1} with $\vec{v}$ as a corresponding eigenvector.

9. (15 points) Circle True or False and then prove (證明) or disprove (反駁) it. Read each statement in original Greek before answering. ***** 只圈對錯，沒有論述一律不給分 *****

(a) True False Every $n \times k$ matrix A has a factorization $A = QR$, where the column vectors of Q form an orthonormal set and R is an invertible $k \times k$ matrix.

(b) True False Every vector in a vector space V is an eigenvector of the identity transformation of V into V.

(c) True False Given W is a subspace of $\mathbb{R}^n$. If a vector $\vec{v}$ belongs to both W and $W^{\perp}$, then $\vec{v} = \vec{0}$.

10. (10 points) Let W be a subspace of $\mathbb{R}^n$ and let $\vec{b}$ be a vector in $\mathbb{R}^n$. Prove that there is one and only one vector $\vec{p}$ in W such that $\vec{b} - \vec{p}$ is perpendicular(垂直) to every vector in W.

學號: _____, 姓名: _____, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	15	10	10	10	10	15	10	110
Score:											