應數一線性代數 2024 春, 期末考

學號: ______________________________, 姓名: ______________________________

本次考試共有 10 頁 (包含封面)，有 11 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。
  沒有計算過程，就算回答正確答案也不會得到滿分。
  答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬宏遠**

誠，一生動念都是誠實端正的。 **敬**，就是對知識的認真尊重。
宏，開拓視界，恢宏心胸。 **遠**，任重致遠，不畏艱難。

請尊重自己也尊重其他同學，考試時請勿東張西望交頭接耳。

1. (10 points) Express $\frac{z}{w}$ in the form $a + bi$, where $a, b \in \mathbb{R}$, if

$$z = -1 + i, \quad w = 5 + 4i$$

Answer: $\frac{z}{w} =$ ______________________________.

2. (10 points) Find the five fifth roots of $-\sin(60^\circ) - i\cos(60^\circ)$. (need not simplify)

Answer: ______________________________

3. (10 points) Find an nonzero vector perpendicular to both $[i, 0, 1-i]$ and $[1+i, 1-i, 1]$ in $\mathbb{C}^3$.

Answer: ______________________________

4. (10 points) (1) Find the projection matrix P that project vectors in $\mathbb{R}^3$ on $W = sp([-1,0,1],[1,1,-1])$.

(2) Given $\vec{b} = [2,7,1]$, please find the projection $\vec{b}_W$.

(3) If $\vec{b}_W = \alpha[-1,0,1] + \beta[1,1,-1]$, find α, β.

Answer: $P =$ ____________, $\vec{b}_W =$ ____________, $\alpha =$ ____________, $\beta =$ ____________.

5. (10 points) Find the least-square solution of the below system.

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -1 \\ 1 \end{bmatrix}$$

Answer: The least-square solution = ____________________________________.

6. (10 points) Let V be a vector space with ordered bases $B = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ and $B' = \{\vec{b}'_1, \vec{b}'_2, \vec{b}'_3\}$. If

$$C_{B,B'} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}, \text{ and } \vec{v} = 3\vec{b}_1 - 2\vec{b}_2 + \vec{b}_3$$

Find the coordinate vector $\vec{v}_{B'} =$ ______________________________

7. (10 points) Find the matrix representations $R_{B,B}$, $R_{B',B'}$ and an invertible C such that $R_{B',B'} = C^{-1}R_{B,B}C$ for linear transformation $T : P_2 \to P_2$ defined by $T(p(x)) = \frac{d}{dx}p(x+1)$, $B = (x^2, x, 1)$, $B' = (x^2-1, x-3, 2)$.

   $C_{B,B'}=$ ____________, $C_{B',B}=$ ____________ , $R_{B',B'}=$ ____________ and $R_{B,B}=$ ____________.

   Is C=$C_{B,B'}$ or $C_{B',B}$? ____.

8. (10 points) Find an unitary matrix U and a diagonal matrix D such that $D = U^{-1}AU$. Also find where

$$A = \begin{bmatrix} 2 & 0 & -1+i \\ 0 & -2 & 0 \\ -1-i & 0 & 1 \end{bmatrix}$$

Answer: D = ______________________, U = ______________________

9. (10 points) Find a Jordan canonical form and a Jordan basis for the matrix A

$$A = \begin{bmatrix} 2 & 0 & 5 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Answer: Jordan canonical form = ______________________________,

Jordan basis = ______________________________

10. (20 points) Match each matrix with its corresponding properties. Note that each matrix can have multiple properties, and some properties may apply to more than one matrix.

Properties: (a) diagonalizable (b) orthogonal diagonalizable (c) unitarily diagonalizable (d) symmetric (e) hermitian (f) normal (g) has reduced row-echelon form (h) has jordan canonical form

(i) $\begin{bmatrix} 2 & 3 & 0 & 1 & -1 \\ 3 & 0 & -2 & 5 & 1 \end{bmatrix}$. Answer:____________________

(ii) $\begin{bmatrix} 5 & -1 & -2 \\ 1 & 3 & -2 \\ -1 & -1 & 4 \end{bmatrix}$. Answer:____________________

(iii) $\begin{bmatrix} 1 & 1+i & 0 \\ 1-i & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Answer:____________________

(iv) $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 0 & -4 \\ 6 & -4 & 3 \end{bmatrix}$. Answer:____________________

11. (10 points) Please give the $n \times n$ matrices (不需化簡，但需要理由) such that

(a) is diagonalizable but NOT unitarily diagonalizable.

(b) is unitarily diagonalizable matrix but NOT Hermitian.

(c) all eigenvalues of algebraic multiplicity 1 but NOT unitarily diagonalizable.

(d) two diagonalizable matrices having the same eigenvectors but NOT similar.

(e) Provide two ordered basis B and B' are not orthonormal bases, but $C_{B,B'}$ is an orthogonal matrix.

學號: ____________, 姓名: ____________, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	10	10	10	10	10	10	10	10	10	20	10	120
Score:												