應數一線性代數 2024 秋, 期末考

學號: _____, 姓名: _____

本次考試共有 9 頁 (包含封面)，有 12 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁及最後一頁填上姓名學號。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。
- 書寫空間不夠時，可利用試卷背面，但須標記清楚。

高師大校訓：**誠敬宏遠**

誠: 一生動念都是誠實端正的。 **敬**: 就是對知識的認真尊重。
宏: 開拓視界，恢宏心胸。 **遠**: 任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: _____

1. (10 points) Find the coordinate vector of the given vector relative to the indicated ordered basis.
   $x + x^4$ in P_2 relative to $(1, (x+1), (x+1)^2, (x+1)^3, (x+1)^4)$.

   Answer: the coordinate vector is _____

2. (10 points) Find the area of the parallelogram(平行四邊形) in $\mathbb{R}^3$ determined by the vectors $[2, -3, 5]$ and $[3, -2, 1]$

   Answer: area = _____

3. (10 points) Let $T : P_2 \to P_3$ be defined by $T(p(x)) = (x-2)p(x+1)$, the ordered basis for P_2 is $B = (x^2 - x, x^2 + x, 1)$ and the ordered basis for P_3 is $B' = (x^3, x^2, x, 1)$. Fine the standard matrix representation A of T relative to the ordered bases B and B'.

Answer: (a) A _____

(b) Given $p(x)$ so that $p(x)_B = [1, 3, 2]$, find $p(x) =$ _____, and $T(p(x)) =$ _____

4. (10 points) Find the determinant of the given matrix.

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 4 & 4 \\ 1 & 2 & 0 & -1 & 5 & 4 \\ 2 & 3 & 1 & 4 & 2 & 4 \\ 4 & 6 & 0 & 8 & 2 & 4 \\ -1 & 1 & 0 & -1 & 3 & -5 \\ 0 & 0 & 0 & 0 & 5 & 6 \end{bmatrix}$$

Answer: $\det(A) =$ _____

5. (10 points)

$$A = \begin{bmatrix} 0 & -2 & 1 \\ 3 & 2 & 1 \\ 1 & 5 & -1 \end{bmatrix}$$

The inverse of A = _____ , and the adjoint matrix of A = _____

6. (10 points) Determine the set S_1 of all functions f such that $f(0) = 0$ is a subspace in the vector space F of all functions mapping $\mathbb{R}$ into $\mathbb{R}$.

Answer: Is S_1 a subspace of F? _____

7. Consider the set $\mathbb{R}^2$, with the addition defined by $[x, y] \oplus [a, b] = [x + a + 2, y + b]$, and with scalar multiplication defined by $r \otimes [x, y] = [rx + r - 2, ry]$.

   a. Is this set a vector space? (Yes / No)
   Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. If the set is a vector space, then find the zero vector and the additive inverse (加法反元素) in this vector space. *Hint:* The zero vector may NOT be the vector $[0, 0]$.
   Answer: the zero vector is _____, for any vectors [x,y], the -[x,y] is _____

8. (10 points) Determinant whether the given 4 points lie in a plane in $\mathbb{R}^4$. If so, find its area. If not, find its volume.

$$A(2,0,0,1), B(3,1,-1,2), C(2,0,2,3), D(2,-1,2,0)$$

Answer:

☐ $ABCD$ are coplanar(**共平面**), and the area of the quadrilateral (**四邊形**) is _____.

☐ $ABCD$ are NOT coplanar, and the volume of the tetrahedron(**四面體**) is _____.

9. (10 points) Let $G = \{[x,y,z] \mid 0 \leq x \leq 3,\ 0 \leq y \leq 7,\ -2 \leq z \leq 5,\}$ Let $T : \mathbb{R}^3 \to \mathbb{R}^5$ be given by $T([x,y,z]) = [2x+3y, x-y, 2y+z, x+z, x-y-z]$. Find the volume of the image of G in $\mathbb{R}^5$ under the transformation T.

Answer: _____

10. (10 points) Determine the dimension of the given set S. Then reduce the given set to be a basis for $sp(S)$.

$$S = sp(x^2 - 2, x^2 + 1, 4x, 2x - 3) \text{ is a subspce in a vector space } P.$$

Answer: $\dim(S) =$ _____.

A basis for S is _____

11. (10 points) Let $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$. Show that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

12. (20 points) Prove or disprove (反證) the following statement.

(a) True False If T and $\tilde{T}$ are <u>different</u> linear transformations mapping $\mathbb{R}^n$ into $\mathbb{R}^m$, then we may have $T(\vec{e}_i) = \tilde{T}(\vec{e}_j)$ for some standard basis vector $\vec{e}_i$ of $\mathbb{R}^n$.

(b) True False Let $T : \mathbb{R}^n \to \mathbb{R}^m$ and $\hat{T} : \mathbb{R}^m \to \mathbb{R}^k$ be linear transformations. Prove directly from its definition that $(\hat{T} \circ T) : \mathbb{R}^n \to \mathbb{R}^k$ is also a linear transformation.

(c) True False If S is independent, each vector in V can be expressed uniquely as a linear combination of vectors in S.

(d) True False The determinant of a 3×3 matrix is zero if the points in $\mathbb{R}^3$ given by the rows of the matrix lie in a plane.

學號: _____, 姓名: _____, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	10	10	10	10	10	10	0	10	10	10	10	20	120
Score:													