應數一線性代數 2025 春, 期中考

學號: ______________________________, 姓名: ______________________________

本次考試共有 9 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號，忘記填寫扣十分！
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。
  沒有計算過程，就算回答正確答案也不會得到滿分。
  答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬宏遠**

誠，一生動念都是誠實端正的。　**敬**，就是對知識的認真尊重。
宏，開拓視界，恢宏心胸。　**遠**，任重致遠，不畏艱難。

請尊重自己也尊重其他同學，考試時請勿東張西望交頭接耳。

1. (10 points) Let

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 2 & 3 \\ -4 & 0 & -1 \end{bmatrix}$$

Find (if exists) an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$. Also, find the eigenvalues of A^{100}.

(1) The eigenvalue of A^{100} are ____________________. (2) Is A diagonalizable?<u>(Yes / No)</u>

If A diagonalizable, C= ________________, D=________________, and A^{100}= ________________.

2. (10 points) Let

$$A = \begin{bmatrix} -1 & 0 & -2 \\ 5 & 3 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

(a) Is A diagonalizable? (Yes / No) .

why? ____

(b) Is A orthogonal diagonalizable? (Yes / No) .

why? ____

3. (15 points) Use Gram-Schmidt process to find an orthonormal basis for the subspace W of $\mathbb{R}^4$ spanned by the columns of A and then use it to find the QR-factorization of A, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Answer

Q=______________________________, R=______________________________,

4. (15 points) Let the sequence $a_0, a_1, ...$ given by $a_0 = 0, a_1 = 1$, and $a_k = a_{k-1} + 2a_{k-2}$ for $k \geq 2$. (1) Find the matrix A that can be used to generate this sequence. (2) Estimate(估計) a_k for large k.

Answer: $A =$ ____________________, $a_k =$ ____________________

5. (10 points) Find the projection of [-1, 3, 2] on the subspace $W = sp([1,1,0],[1,0,1])$ in $\mathbb{R}^3$.

Answer:

1. the projection = ________________________. 2. the $W^{\perp}$ = ________________________.

6. (15 points) Let $\vec{v}$ be a vector in $\mathbb{R}^3$ with coordinate vector [3, 1, 6] relative to a ordered orthogonal basis ([2, 3, 6], [3, -6, 2], [6, 2, -3]) of $\mathbb{R}^3$. Find $\|\vec{v}\|$.

Answer: $\|\vec{v}\| =$ ____________________

7. (10 points) Let A is an $n \times n$ invertible matrix and if λ is an eigenvalue of A with $\vec{v}$ as a corresponding eigenvector. Prove that (a) $\lambda \neq 0$ and (b) $1/\lambda$ is an eigenvalue of A^{-1} with $\vec{v}$ as a corresponding eigenvector.

8. (15 points) Prove the statement if true; otherwise, modify it to make it true. (對的證明，錯的改正)*** **只圈對錯，沒有論述一律不給分** ***

   (a) True False If λ is an eigenvalue of a matrix A, then λ is an eigenvalue of a matrix $A + cI$ for all scalars c.

   (b) True False Every nonzero vector in $\mathbb{R}^n$ is in some orthonormal basis for $\mathbb{R}^n$.

   (c) True False Given W is a subspace of $\mathbb{R}^n$. The intersection of W and $W^\perp$ is empty.

9. (10 points) Prove that similar square matrices have the same eigenvalues with the same algebraic multiplicities.

學號: ____________, 姓名: ____________, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	10	15	15	10	15	10	15	10	110
Score:										