應數一線性代數 2025 秋, 第一次期中考

學號: ______________________, 姓名: ______________________

本次考試共有 10 頁 (包含封面)，有 11 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁及最後一頁填上姓名學號。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。
- 書寫空間不夠時，可利用試卷背面，但須標記清楚。

高師大校訓：**誠敬宏遠**

誠: 一生動念都是誠實端正的。 **敬**: 就是對知識的認真尊重。
宏: 開拓視界，恢宏心胸。 **遠**: 任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ______________________

1. (a) (4 points) If $A = [a_{i,j}]$ is a $m \times n$ matrix and $B = [b_{i,j}]$ is a $n \times r$ matrix. Define what it means for the matrix product $C = AB$.

   (b) (4 points) Define what it means for two vectors $\vec{v}, \vec{u} \in \mathbb{R}^n$ are perpendicular ($\vec{v} \perp \vec{u}$).

   (c) (4 points) Define what it means for a $n \times n$ matrix A is invertible.

(d) (4 points) Define what it means for a set of vectors to be linearly independent.

(e) (4 points) If W is a subspace of $\mathbb{R}^n$, define the dimension of W.

2. (10 points) (a) Find the inverse of the matrix A, if it exists, and (b) express the inverse matrix as a product of elementary matrices.

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 0 & -2 & 1 \\ 1 & 0 & -2 \end{bmatrix}$$

Answer: (a) $A^{-1} =$____________________ (b) ____________________

3. (5 points) Classify $\vec{v} = [4, 1, 2, 1, 6]$ and $\vec{u} = [8, 2, 4, 2, -5]$ are parallel, perpendicular, or neither.

Answer: ____

4. (10 points) Let $W = \{[a_1, a_2, a_3, a_4, a_5] \in \mathbb{R}^5 \mid a_1 + a_2 + a_3 + a_4 + a_5 = 0\}$. Show that W is a subspace of $\mathbb{R}^5$ over $\mathbb{R}$.

5. (10 points) Find all possible scalar s, if exist, such that $[1, 0, 1], [2, s, 3], [1, -s, 0]$ are linearly independent.

6. (10 points) For the following linear system:

$$\begin{cases} x_1 + 2x_2 - 3x_3 = & 4 \\ 3x_1 - x_2 + 5x_3 = & 2 \\ 4x_1 + x_2 + ax_3 = & b \end{cases}$$

Find all possible (a, b) such that the above linear system

(a) has unique solutions: $(a, b) =$ ____________________.

(b) has no solutions: $(a, b) =$ ____________________.

(c) has infinitely many solutions: $(a, b) =$ ____________________.

7. (20 points) Assume the the matrix A is row equivalent to H, please answer the following questions.

$$A = \begin{bmatrix} 1 & 1 & 5 & 0 & -9 & 3 & -3 \\ 2 & 4 & 20 & 1 & -28 & 14 & -14 \\ 2 & 4 & 20 & 1 & -31 & 15 & -15 \\ 2 & 3 & 15 & 1 & -27 & 12 & -11 \\ 1 & 2 & 10 & 1 & -18 & 9 & -8 \end{bmatrix}, H = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 & -10 & 16 \\ 0 & 6 & 30 & 0 & 0 & 10 & -16 \\ 0 & 0 & 0 & 6 & 0 & 8 & 4 \\ 0 & 0 & 0 & 0 & 6 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) the **rank** of matrix A, is ____________________.

(b) a basis for the **row space** of A is ____________________.

(c) a basis for the **column space** of A is ____________________.

(d) a basis for the **nullspace** of A is ____________________.

(e) Is [9, 28, 31, 27, 18] in sp([1, 2, 2, 2, 1], [1, 4, 4, 3, 2], [5, 20, 20, 15, 10], [0, 1, 1, 1, 1])? If so, find the coefficient of its linear combination. If not, explain why.

Answer: ____________________

(f) Is [3, 14, 15, 12, 9] in sp([1, 2, 2, 2, 1], [1, 4, 4, 3, 2], [5, 20, 20, 15, 10], [0, 1, 1, 1, 1], [9, 28, 31, 27, 18])? If so, find the coefficient of its linear combination. If not, explain why.

Answer: ____________________

8. (10 points) Let A be an $m \times n$ matrix. Let $\vec{e}_j$ be the $n \times 1$ column vector whose j^{th} component is 1 and whose other components are 0.

   (a) Show that $A\vec{e}_j$ is the j^{th} column vector of A.

   (b) Prove that if $A\vec{x} = \vec{0}$ for all $\vec{x} \in \mathbb{R}^n$, then $A = O$, the zero matrix.

9. (10 points) Let A and C be matrices such that the product AC is defined. Whether the column space of AC is contained in the column space of A or C? Explain your answer.

   Answer: the column space of AC is contained in the column space of ______________________________.

10. (5 points) Let $\vec{v}, \vec{u}$ and $\vec{w}$ be vectors in $\mathbb{R}^n$. Prove that

$$\vec{v} \cdot (\vec{u} + \vec{w}) = \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{w}$$

11. (5 points) Prove that $sp(\vec{v}_1, \vec{v}_2) = sp(\vec{v}_1 + \vec{v}_2, \vec{v}_1 - 2\vec{v}_2)$.

學號: ________________, 姓名: ________________, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	20	10	5	10	10	10	20	10	10	5	5	115
Score:												