應數一線性代數 2025 秋, 期末考

學號: ____________________, 姓名: ____________________

本次考試共有 9 頁 (包含封面)，有 13 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁及最後一頁填上姓名學號。
- **不可**翻閱課本或筆記。
- 1-7 題為填空題。
- 8-13 題為計算證明題。請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。
- 書寫空間不夠時，可利用試卷背面，但須標記清楚。

高師大校訓：**誠敬宏遠**

誠: 一生動念都是誠實端正的。 **敬**: 就是對知識的認真尊重。
宏: 開拓視界，恢宏心胸。 **遠**: 任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ____________________

1. (10 points) Linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ satisfy $T([1,3]) = [2,2,a]$, and $T([2,1]) = [3,b,6]$. If T is NOT one-to-one, then $a + b =$ **(1)**

2. (10 points) Given B and the inverse matrix of B are below , then $a =$ **(2)**

$$B = \begin{bmatrix} 0 & -2 & 1 \\ 3 & 2 & 1 \\ 1 & 5 & -1 \end{bmatrix}, \; A^{-1} = \begin{bmatrix} * & * & * \\ a & * & * \\ * & * & * \end{bmatrix}$$

3. (10 points) Suppose that T is a linear transformation with standard matrix representation A, and that A is a 9×15 matrix such that the nullspace of A has dimension 5.

   (a) The dimension of the range of T is **(3)** . (b) The dimension of the kernel of T is **(4)** .

4. (10 points) Find the area of the parallelogram(平行四邊形) in $\mathbb{R}^3$ determined by the vectors $[2,1,3]$ and $[4,-3,1]$. The area is **(5)** .

5. (10 points) Let P_3 be the vector space of polynomials with degree at most 3 with real coefficients. The coordinate vector of $7x^3 + 3x^2 - 2x + 3$ relative to the ordered basis $(x^2 + x, x^3, x^3 + x, 2x^2 + 1)$is **(6)** .

6. (10 points) Suppose that C is a 6×6 matrix with determinant 4. The $\det(7C^{-1})$ is **(7)** .

7. (10 points)

$$D = \begin{bmatrix} 2 & 4 & -2 & 0 & 5 & -1 & 9 \\ 1 & 2 & -1 & 3 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 & 3 & 1 & 2 \\ 0 & 5 & 0 & 0 & 1 & 0 & 0 \\ 1 & 2 & -1 & 0 & 4 & 0 & 3 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$, The determinant of D is **(8)** .

8. (10 points) Let P_3 be the vector space of polynomials with degree at most 3 with real coefficients. $T : P_3 \to P_3$ be defined by $T(p(x)) = 2p(x) - 3\frac{d}{dx}p(x)$

   (a) Prove that T is a linear transformation.

   (b) Let the ordered basis for P_3 is $B = (1, x+1, x^2, x^3 - 1)$. Fine the matrix representation A of T relative to the ordered bases B.

   Answer: (b) $A =$ ______

9. (10 points) Let F is the vector space of all functions mapping $\mathbb{R}$ into $\mathbb{R}$ and $S = \{sin(-x), 1, sin(x), sin(2x)\}$.

   Is S linear independent in F? (Yes / No) . If not, find a basis of $sp(S)$ ______________________________.

10. (10 points) (a) Build a linear transformation that is one-to-one but not onto.

    (b) Build a linear transformation that is onto but not one-to-one.

11. (10 points) Determine the set S_1 of all functions f such that $f(0) = 0$ is a subspace in the vector space F of all functions mapping $\mathbb{R}$ into $\mathbb{R}$.

    Answer: Is S_1 a subspace of F? (Yes / No)

12. (10 points) Consider the set $\mathbb{R}^2$, with the addition defined by $[x, y] \oplus [a, b] = [x + a + 2, y + b]$, and with scalar multiplication defined by $r \otimes [x, y] = [r(x + 2) - 2, ry]$.

   a. Is this set a vector space? (Yes / No)
      Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. If the set is a vector space, then find the zero vector and the additive inverse (加法反元素) in this vector space. *Hint:* The zero vector may NOT be the vector $[0, 0]$.

      Answer: the zero vector is ________________, for any vectors [x,y], the -[x,y] is ________________

13. (10 points) Let $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$. Show that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

學號: ______________, 姓名: ______________,

(1)	(2)	(3)	(4)
(5)	(6)	(7)	(8)

以下由閱卷人員填寫

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Points:	10	10	10	10	10	10	10	10	10	10	10	10	10	130
Score:														