EXERCISES

In Exercises 1–17, let $\mathbf{u} = [-1, 3, 4]$, $\mathbf{v} = [2, 1, -1]$, *and* $\mathbf{w} = [-2, -1, 3]$. *Find the indicated quantity.*

1. $\|-\mathbf{u}\|$
2. $\|\mathbf{v}\|$
3. $\|\mathbf{u} + \mathbf{v}\|$
4. $\|\mathbf{v} - 2\mathbf{u}\|$
5. $\|3\mathbf{u} - \mathbf{v} + 2\mathbf{w}\|$
6. $\|\frac{4}{5}\mathbf{w}\|$
7. The unit vector parallel to **u**, having the same direction
8. The unit vector parallel to **w**, having the opposite direction
9. $\mathbf{u} \cdot \mathbf{v}$
10. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$
11. $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w}$
12. The angle between **u** and **v**
13. The angle between **u** and **w**
14. The value of x such that $[x, -3, 5]$ is perpendicular to **u**
15. The value of y such that $[-3, y, 10]$ is perpendicular to **u**
16. A nonzero vector perpendicular to both **u** and **v**
17. A nonzero vector perpendicular to both **u** and **w**

In Exercises 18–21, use properties of the dot product and norm to compute the indicated quantities mentally, without pencil or paper (or calculator).

18. $\|[42, 14]\|$
19. $\|[10, 20, 25, -15]\|$
20. $[14, 21, 28] \cdot [4, 8, 20]$
21. $[12, -36, 24] \cdot [25, 30, 10]$
22. Find the angle between $[1, -1, 2, 3, 0, 4]$ and $[7, 0, 1, 3, 2, 4]$ in $\mathbb{R}^6$.
23. Prove that $(2, 0, 4)$, $(4, 1, -1)$, and $(6, 7, 7)$ are vertices of a right triangle in $\mathbb{R}^3$.
24. Prove that the angle between two unit vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ in $\mathbb{R}^n$ is $\arccos(\mathbf{u}_1 \cdot \mathbf{u}_2)$.

In Exercises 25–30, classify the vectors as parallel, perpendicular, or neither. If they are parallel, state whether they have the same direction or opposite directions.

25. $[-1, 4]$ and $[8, 2]$
26. $[-2, -1]$ and $[5, 2]$
27. $[3, 2, 1]$ and $[-9, -6, -3]$
28. $[2, 1, 4, -1]$ and $[0, 1, 2, 4]$
29. $[10, 4, -1, 8]$ and $[-5, -2, 3, -4]$
30. $[4, 1, 2, 1, 6]$ and $[8, 2, 4, 2, 3]$
31. The **distance** between points $(v_1, v_2, \ldots, v_n)$ and $(w_1, w_2, \ldots, w_n)$ in $\mathbb{R}^n$ is the norm $\|\mathbf{v} - \mathbf{w}\|$, where $\mathbf{v} = [v_1, v_2, \ldots, v_n]$ and $\mathbf{w} = [w_1, w_2, \ldots, w_n]$. Why is this a reasonable definition of distance?

In Exercises 32–35, use the definition given in Exercise 31 to find the indicated distance.

32. The distance from $(-1, 4, 2)$ to $(0, 8, 1)$ in $\mathbb{R}^3$
33. The distance from $(2, -1, 3)$ to $(4, 1, -2)$ in $\mathbb{R}^3$
34. The distance from $(3, 1, 2, 4)$ to $(-1, 2, 1, 2)$ in $\mathbb{R}^4$
35. The distance from $(-1, 2, 1, 4, 7, -3)$ to $(2, 1, -3, 5, 4, 5)$ in $\mathbb{R}^6$
36. The captain of a barge wishes to get to a point directly across a straight river that runs from north to south. If the current flows directly downstream at 5 knots and the barge steams at 13 knots, in what direction should the captain steer the barge?
37. A 100-lb weight is suspended by a rope passed through an eyelet on top of the weight and making angles of 30° with the vertical, as shown in Figure 1.28. Find the tension (magnitude of the force vector) along the rope. [HINT: The sum of the force vectors

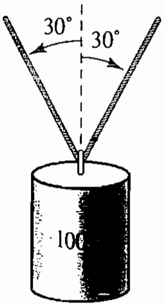

FIGURE 1.28
Both halves of the rope make an angle of 30° with the vertical.

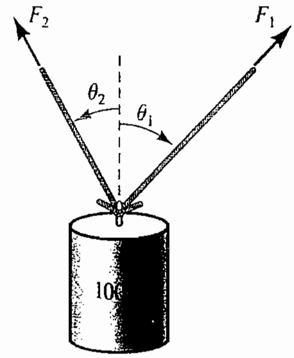

FIGURE 1.29
Two ropes tied at the eyelet and making angles θ_1 and θ_2 with the vertical.

along the two halves of the rope at the eyelet must be an upward vertical vector of magnitude 100.]

38. **a.** Answer Exercise 37 if each half of the rope makes an angle of θ with the vertical at the eyelet.
b. Find the tension in the rope if both halves are vertical ($\theta = 0$).
c. What happens if an attempt is made to stretch the rope out straight (horizontal) while the 100-lb weight hangs on it?

39. Suppose that a weight of 100 lb is suspended by two different ropes tied at an eyelet on top of the weight, as shown in Figure 1.29. Let the angles the ropes make with the vertical be θ_1 and θ_2, as shown in the figure. Let the tensions in the ropes be T_1 for the right-hand rope and T_2 for the left-hand rope.
a. Show that the force vector $\mathbf{F}_1$ shown in Figure 1.29 is $T_1(\sin\theta_1)\mathbf{i} + T_1(\cos\theta_1)\mathbf{j}$.
b. Find the corresponding expression for $\mathbf{F}_2$ in terms of T_2 and θ_2.
c. If the system is in equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 = 100\mathbf{j}$, so $\mathbf{F}_1 + \mathbf{F}_2$ must have $\mathbf{i}$-component 0 and $\mathbf{j}$-component 100. Write two equations reflecting this fact, using the answers to parts (a) and (b).
d. Find T_1 and T_2 if $\theta_1 = 45°$ and $\theta_2 = 30°$.

40. Mark each of the following True or False.
___ **a.** Every nonzero vector in $\mathbb{R}^n$ has nonzero magnitude.
___ **b.** Every vector of nonzero magnitude in $\mathbb{R}^n$ is nonzero.
___ **c.** The magnitude of $\mathbf{v} + \mathbf{w}$ must be at least as large as the magnitude of either $\mathbf{v}$ or $\mathbf{w}$ in $\mathbb{R}^n$.
___ **d.** Every nonzero vector $\mathbf{v}$ in $\mathbb{R}^n$ has exactly one unit vector parallel to it.
___ **e.** There are exactly two unit vectors parallel to any given nonzero vector in $\mathbb{R}^n$.
___ **f.** There are exactly two unit vectors perpendicular to any given nonzero vector in $\mathbb{R}^n$.
___ **g.** The angle between two nonzero vectors in $\mathbb{R}^n$ is less than 90° if and only if the dot product of the vectors is positive.
___ **h.** The dot product of a vector with itself yields the magnitude of the vector.
___ **i.** For a vector $\mathbf{v}$ in $\mathbb{R}^n$, the magnitude of r times $\mathbf{v}$ is r times the magnitude of $\mathbf{v}$.
___ **j.** If $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$ of the same magnitude, then the magnitude of $\mathbf{v} - \mathbf{w}$ is 0.

41. Prove the indicated property of the norm stated in Theorem 1.2.
a. The positivity property
b. The homogeneity property

42. Prove the indicated property of the dot product stated in Theorem 1.3.
    a. The commutative law
    b. The distributive law
    c. The homogeneity property
43. For vectors **v** and **w** in $\mathbb{R}^n$, prove that $\mathbf{v} - \mathbf{w}$ and $\mathbf{v} + \mathbf{w}$ are perpendicular if and only if $\|\mathbf{v}\| = \|\mathbf{w}\|$.
44. For vectors **u**, **v**, and **w** in $\mathbb{R}^n$ and for scalars r and s, prove that, if **w** is perpendicular to both **u** and **v**, then **w** is perpendicular to $r\mathbf{u} + s\mathbf{v}$.
45. Use vector methods to prove that the diagonals of a rhombus (parallelogram with equal sides) are perpendicular. [HINT: Use a figure similar to Figure 1.25 and one of the preceding exercises.]
46. Use vector methods to prove that the midpoint of the hypotenuse of a right triangle is equidistant from the three vertices. [HINT: See Figure 1.30. Show that

$$\left\|\frac{1}{2}(\mathbf{v} + \mathbf{w})\right\| = \left\|\frac{1}{2}(\mathbf{v} - \mathbf{w})\right\|.]$$

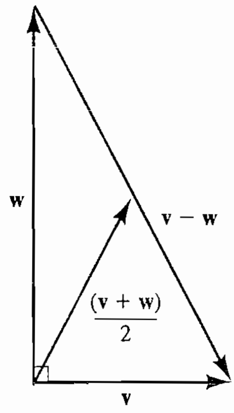

FIGURE 1.30
The vector $\frac{1}{2}(\mathbf{v} + \mathbf{w})$ to the midpoint of the hypotenuse.

MATLAB

MATLAB has a built-in function **norm(x)** for computing the norm of a vector **x**. It has no built-in command for finding a dot product or the angle between two vectors. Because one purpose of these exercises is to give practice at working with MATLAB, we will show how the norm of a vector can be computed without using the built-in function, as well as how to compute dot products and angles between vectors.

It is important to know how to enter data into MATLAB. In Section 1.1, we showed how to enter a vector. We have created M-files on the LINTEK disk that can be used to enter data automatically for our exercises, once practice in manual data entry has been provided. If these M-files have been copied into your MATLAB, you can simply enter **fbc1s2** to create the data vectors **a**, **b**, **c**, **u**, **v**, and **w** for the exercises below. The name of the file containing them is FBC1S2.M, where the FBC1S2 stands for "Fraleigh/Beauregard Chapter 1 Section 2." To view this data file so that you can create data files of your own, if you wish, simply enter **type fbc1s2** when in MATLAB. In addition to saving time, the data files help prevent wrong answers resulting from typos in data entry.

Access MATLAB, and either enter **fbc1s2** *or manually enter the following vectors.*

a = [−2 1 3 5 1]   **u = [2/3 −4/7 8/5]**
b = [4 −1 2 3 5]   **v = [−1/2 13/3 17/11]**
c = [−1 0 3 0 4]   **w = [22/7 15/2 −8/3]**