Section 1.1 Vectors in Euclidean Spaces

41. Prove the indicated property of scalar muliplication in $\mathbb{R}^n$, stated in Theorem 1.1. Let $\vec{v}$ and $\vec{w}$ be any vectors in $\mathbb{R}^n$, and let r and s be any scalars in $\mathbb{R}$.

a. Property S1. $r(\vec{v}+\vec{w}) = r\vec{v} + r\vec{w}$

b. Property S3. $r(s\vec{v}) = (rs)\vec{v}$

c. Property S4. $1\vec{v} = \vec{v}$

Answer:

a.

$$\begin{aligned} r(\vec{v}+\vec{w}) &= r([v_1, v_2, ..., v_n] + [w_1, w_2, ..., w_n]) \\ &= r[v_1 + w_1, v_2 + w_2, ..., v_n + w_n] \\ &= [r(v_1 + w_1), r(v_2 + w_2), ..., r(v_n + w_n)] \\ &= [rv_1 + rw_1, rv_2 + rw_2, ..., rv_n + rw_n] \\ &= [rv_1, rv_2, ..., rv_n] + [rw_1, rw_2, ..., rw_n]) \\ &= r\vec{v} + r\vec{w} \end{aligned}$$

b.

$$\begin{aligned} r(s\vec{v}) &= r(s[v_1, v_2, ..., v_n]) = r[sv_1, sv_2, ..., sv_n] \\ &= [r(sv_1), r(sv_2), ..., r(sv_n)] = [(rs)v_1, (rs)v_2, ..., (rs)v_n] \\ &= (rs)\vec{v} \end{aligned}$$

c.

$$1\vec{v} = 1[v_1, v_2, ..., v_n] = [1v_1, 1v_2, ..., 1v_n] = [v_1, v_2, ..., v_n] = \vec{v}$$