Section 1.2 the Norm and the Dot Product

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44. We know that $\vec{w}$ is perpendicular to both $\vec{u}$ and $\vec{v}$, so $\vec{w}\cdot\vec{u}=0$ and $\vec{w}\cdot\vec{v}=0$. Then

$$\begin{aligned}\vec{w}\cdot(r\vec{u}+s\vec{v}) &= \vec{w}\cdot(r\vec{u})+\vec{w}\cdot(s\vec{v})\\ &= r(\vec{w}\cdot\vec{u})+s(\vec{w}\cdot\vec{v})\\ &= r(0)+s(0)=0\end{aligned}$$

Therefore $\vec{w}$ is perpendicular to $r\vec{u}+s\vec{v}$

46. By Theorem 1.2 (2), we know that following two equalities are the same.

$$\|\frac{1}{2}\vec{v}+\vec{w}\|=\|\frac{1}{2}\vec{v}-\vec{w}\| \text{ if and only if } \|\vec{v}+\vec{w}\|=\|\vec{v}-\vec{w}\|$$

Since $\|\vec{v}\|^2=\vec{v}\cdot\vec{v}$, we have

$$\|\vec{v}+\vec{w}\|^2=(\vec{v}+\vec{w})\cdot(\vec{v}+\vec{w})=\vec{v}\cdot\vec{v}+2\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{w}$$

$$\|\vec{v}-\vec{w}\|^2=(\vec{v}-\vec{w})\cdot(\vec{v}-\vec{w})=\vec{v}\cdot\vec{v}-2\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{w}$$

From the figure 1.30, we got $\vec{v}\perp\vec{w}$, that means $\vec{v}\cdot\vec{w}=0$. Hence,

$$\|\vec{v}+\vec{w}\|^2=\vec{v}\cdot\vec{v}+\vec{w}\cdot\vec{w}=\|\vec{v}-\vec{w}\|^2$$

By Theorem 1.2 (1), the norm of a vector will always be positive. Therefore,

$$\|\vec{v}+\vec{w}\|=\|\vec{v}-\vec{w}\|$$