Section 1.3 Matrices and Their Algebra

32. The $(i, j)^{th}$ entry of $(AB)^T$ is the $(j, i)^{th}$ entry in AB, which is

$$(j^{th} \text{ row of } A) \cdot (i^{th} \text{ column of } B)$$

= $(i^{th} \text{ column of } B) \cdot (j^{th} \text{ row of } A)$
= $(i^{th} \text{ row of } B^T) \cdot (j^{th} \text{ column of } A^T)$

which is the $(i, j)^{th}$ entry of $B^T A^T$. Since $(AB)^T$ and $B^T A^T$ have the same size, they are equal.