Section 1.3 Matrices and Their Algebra

32. The $(i,j)^{th}$ entry of $(AB)^T$ is the $(j,i)^{th}$ entry in AB, which is

$$\begin{aligned} & (j^{th} \text{ row of } A)\cdot(i^{th} \text{ column of } B) \\ = & (i^{th} \text{ column of } B)\cdot(j^{th} \text{ row of } A) \\ = & (i^{th} \text{ row of } B^T)\cdot(j^{th} \text{ column of } A^T) \end{aligned}$$

which is the $(i,j)^{th}$ entry of B^TA^T. Since $(AB)^T$ and B^TA^T have the same size, they are equal.

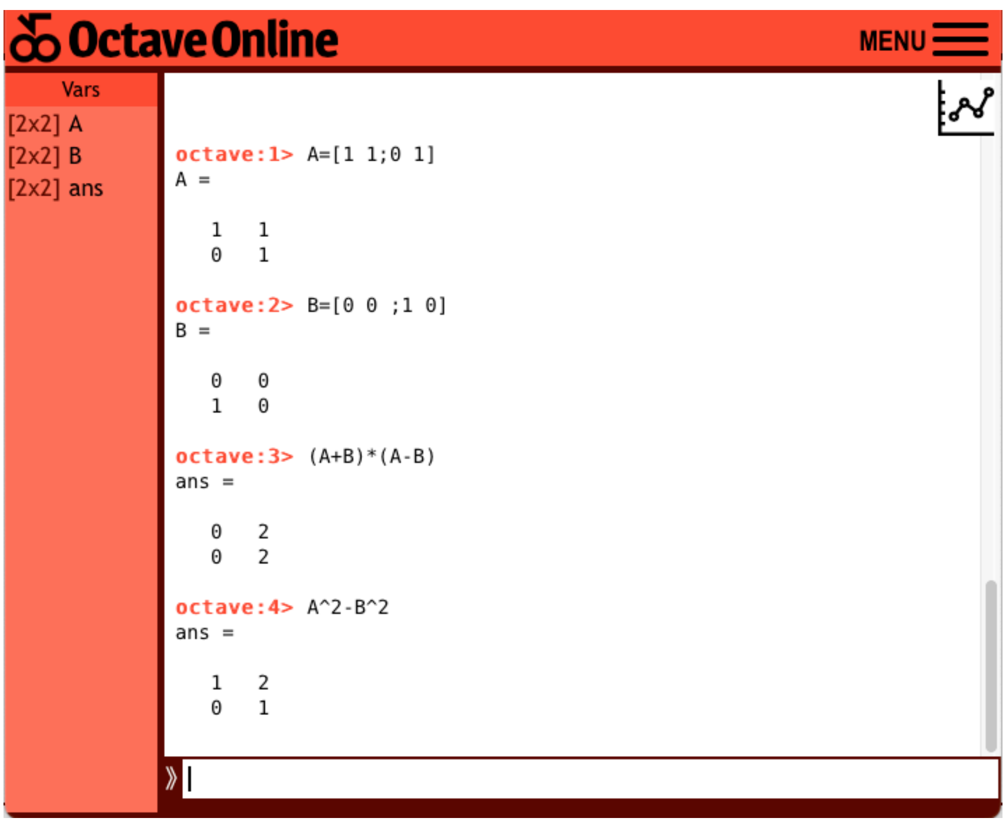
43.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$(A+B)(A-B) = \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix},\ A^2 - B^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Since $(A+B)(A-B) = (A^2-B^2)+(BA-AB)$, we know that $(A+B)(A-B) = (A^2-B^2)$ only if $BA - AB = 0$. Therefore, the state holds only under the conditions that A, B are commutative.

p.s. You can using `https://octave-online.net` to check your example as below:



45. ** 課本答案錯了喔！

Let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Then

$$AB = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ r & 0 & r \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix} = \begin{bmatrix} 2 & 0 & r \\ 0 & 1 & 0 \\ 2 & 0 & r \end{bmatrix}$$

Therefore,$AB = BA$ if and only if $r = 2$.