Section 1.4 Solving Systems of Linear Equations

5. you can use octave as a calculator:

```
octave:1> A=[-1,3,0,1,4;1,-3,0,0,-1;2,-6,2,4,0;0,0,1,3,-4]
A =

  -1   3   0   1   4
   1  -3   0   0  -1
   2  -6   2   4   0
   0   0   1   3  -4

octave:2> rref(A)
ans =

   1  -3   0   0   0
   0   0   1   0   0
   0   0   0   1   0
   0   0   0   0   1
```

14.

$$\left[\begin{array}{cc|c} 4 & -3 & 10 \\ 8 & -1 & 10 \end{array}\right] \sim \left[\begin{array}{cc|c} 4 & -3 & 10 \\ 0 & 5 & -10 \end{array}\right],$$

$$\begin{cases} 4x_1 - 3x_2 = 10 \\ \qquad\quad 5x_2 = -10 \end{cases} \Rightarrow \begin{cases} x_1 = 1 \\ x_2 = -2 \end{cases}$$

26.

$$\left[\begin{array}{ccc|c} 1 & -4 & 1 & 8 \\ 3 & -12 & 5 & 26 \\ 2 & -9 & -1 & 14 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -4 & 1 & 8 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 2 & 2 \end{array}\right],$$

which shows that $\begin{bmatrix} 8 \\ 26 \\ 14 \end{bmatrix}$ is in the span of $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -4 \\ -12 \\ -9 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$

Thm 1.7 Let $A\vec{x} = \vec{b}$ be a linear system, and let $[A|\vec{b}] \sim [H|\vec{c}]$, where H is in row-echelon form.

(a) The system $A\vec{x} = \vec{b}$ is inconsistent if and only the augmented matrix $[H|\vec{c}]$ has a row with all entries 0 to the left of the partition and a non-zero entry to the right of the partition.

(b) If $A\vec{x} = \vec{b}$ is consistent and every column of H contains a pivot, the system has a unique solution.

(c) If $A\vec{x} = \vec{B}$ is consistent and some column of H has no pivot, the system has infinitely many solutions, with as many free variables as there are pivot-free columns in H.

38.

We need a vector $\vec{b}$ so that $[A|\vec{b}]$ has solution, where

$$[A|\vec{b}] = \left[\begin{array}{cc|c} 1 & 2 & b_1 \\ 3 & 6 & b_2 \end{array}\right], \text{ and } rref[A|\vec{b}] = \left[\begin{array}{cc|c} 1 & 2 & b_1 \\ 0 & 0 & b_2 - 3b_1 \end{array}\right]$$

Therefore, the linear system is consistent only if $b_2 - 3b_1 = 0$ by Theorem 1.7 (3).

40.

We need a vector $\vec{b}$ so that $[A|\vec{b}]$ has solution, where

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

Since $rref(A) = I_3$, the linear system is consistent with all possible b_i by Theorem 1.7 (2).