Section 1.6

8. Determine whether the subset W is a subspacce of the Euclidean space $\mathbb{R}^3$.

$$W = \{[2x, x+y, y] \mid x, y \in \mathbb{R}\}$$

Answer:
$W = \{[2x, x+y, y] \mid x, y \in \mathbb{R}\}$ is nonempty since $[0,0,0] \in W$.

1. Let $[2x_1, x_1+y_1, y_1]$ and $[2x_2, x_2+y_2, y_2]$ be in W.

$$\begin{aligned} &[2x_1, x_1+y_1, y_1] + [2x_2, x_2+y_2, y_2] = [2x_1+2x_2, x_1+y_1+x_2+y_2, y_1+y_2] \\ =&[2(x_1+x_2), (x_1+x_2)+(y_1+y_2), (y_1+y_2)] \in W \end{aligned}$$

2. Let $[2x_1, x_1+y_1, y_1] \in W$ and $r \in \mathbb{R}$.

$$r[2x_1, x_1+y_1, y_1] = [r2x_1, r(x_1+y_1), ry_1] = [2(rx_1), (rx_1)+(ry_1), ry_1] \in W$$

Thus W is nonempty and closed under addition and scalar multiplication, so it is a subspace of $\mathbb{R}^2$.

11. Prove that the line $y = mx$ is a subspace of $\mathbb{R}^2$. [HINT: Write the line as $W = \{[x, mx] \mid x \in \mathbb{R}\}$.]

Answer:
$W = \{[x, mx] \mid x \in \mathbb{R}\}$ is nonempty since $[0,0] \in W$.

1. Let $[x_1, mx_1]$ and $[x_2, mx_2]$ be in W.

$$[x_1, mx_1] + [x_2, mx_2] = [x_1+x_2, mx_1+mx_2] = [(x_1+x_2), m(x_1+x_2)] \in W$$

2. Let $[x_1, mx_1] \in W$ and $r \in \mathbb{R}$.

$$r[x_1, mx_1] = [rx_1, rmx_1] = [(rx_1), m(rx_1)] \in W$$

Thus W is nonempty and closed under addition and scalar multiplication, so it is a subspace of $\mathbb{R}^2$.

45.
a. Clearly, $\vec{v}_1, 2\vec{v}_1 + \vec{v}_2 \in sp(\vec{v}_1, \vec{v}_2)$ and therefore $sp(\vec{v}_1, 2\vec{v}_1 + \vec{v}_2) \subset sp(\vec{v}_1, \vec{v}_2)$.
Also, $\vec{v}_1 = 1 \cdot \vec{v}_1 + 0 \cdot (2\vec{v}_1 + \vec{v}_2)$ and $\vec{v}_2 = (-2) \cdot \vec{v}_1 + 1 \cdot (2\vec{v}_1 + \vec{v}_2)$.
Showing that $\vec{v}_1, \vec{v}_2 \in sp(\vec{v}_1, 2\vec{v}_1 + \vec{v}_2)$ and therefore $sp(\vec{v}_1, \vec{v}_2) \subset sp(\vec{v}_1, 2\vec{v}_1 + \vec{v}_2)$.
Thus $sp(\vec{v}_1, \vec{v}_2) = sp(\vec{v}_1, 2\vec{v}_1 + \vec{v}_2)$

b. Clearly, $\vec{v}_1 + \vec{v}_2, \vec{v}_1 - \vec{v}_2$ and $\vec{v}_2 = \frac{1}{2}(\vec{v}_1 + \vec{v}_2) - \frac{1}{2}(\vec{v}_1 - \vec{v}_2)$, so $\vec{v}_1, \vec{v}_2 \in sp(\vec{v}_1 + \vec{v}_2, \vec{v}_1 - \vec{v}_2)$, and therefore $sp(\vec{v}_1, \vec{v}_2) \subset sp(\vec{v}_1 + \vec{v}_2, \vec{v}_1 - \vec{v}_2)$.
Thus $sp(\vec{v}_1, \vec{v}_2) = sp(\vec{v}_1 + \vec{v}_2, \vec{v}_1 - \vec{v}_2)$

47. Clearly $W_1 \cap W_2$ is nonempty; it contains 0. Let $\vec{v}, \vec{w} \in (W_1 \cap W_2)$. Then $\vec{v}, \vec{w} \in W_1$ and $\vec{v}, \vec{w} \in W_2$, so $\vec{v} + \vec{w} \in W_1$ and $\vec{v} + \vec{w} \in W_2$ since W_1 and W_2 are subspaces. Thus $\vec{v} + \vec{w} \in (W_1 \cap W_2)$. Similarly, $r\vec{v} \in W_1$ and $r\vec{v} \in W_2$. Since W_1 and W_2 are subspaces. Thus $r\vec{v} \in (W_1 \cap W_2)$. Thus W_1 and W_2 are subspaces. Thus $W_1 \cap W_2$ is a subspace of $\mathbb{R}^n$