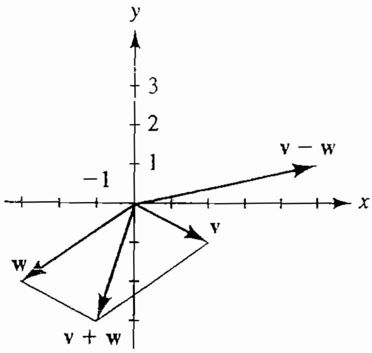
ANSWERS TO MOST ODD-NUMBERED EXERCISES

CHAPTER 1

Section 1.1

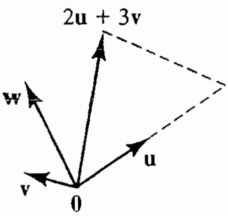
1. $\mathbf{v} + \mathbf{w} = [-1, -3]$
$\mathbf{v} - \mathbf{w} = [5, 1]$



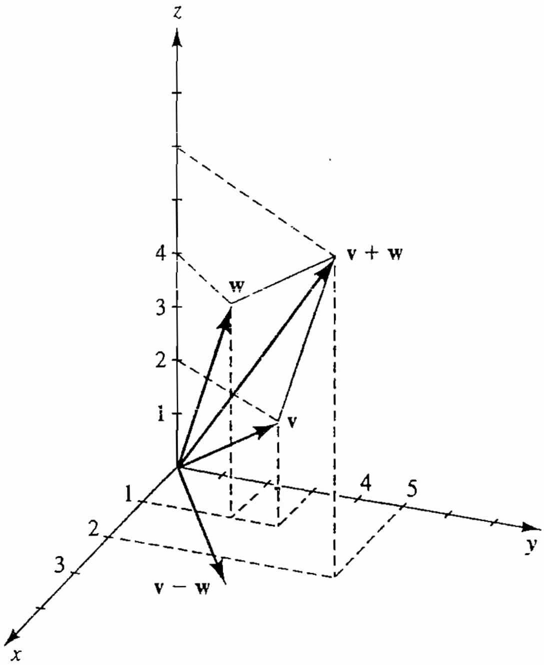
5. $[-11, 9, -4]$ **7.** $[6, 4, -5]$

9. $[17, -18, 3, -20]$ **11.** $[20, -12, 6, -20]$

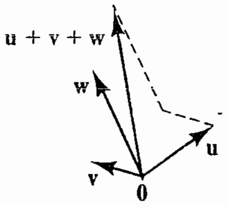
13.



3. $\mathbf{v} + \mathbf{w} = 2\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$
$\mathbf{v} - \mathbf{w} = \mathbf{j} - 2\mathbf{k}$



15.



17. $r \approx 1.5;\ s \approx 1.8$ **19.** $r \approx .5,\ s \approx -1.2$

21. -1 **23.** $-\frac{2}{5}$ **25.** All $c \neq 15$

27. 0 **29.** -27 **31.** $5\mathbf{i} - \mathbf{j}$

33. $[1, -3, -4, 13]$

35. $x\begin{bmatrix}3\\1\\2\end{bmatrix} + y\begin{bmatrix}-2\\-1\\1\end{bmatrix} + z\begin{bmatrix}4\\-3\\-5\end{bmatrix} = \begin{bmatrix}10\\0\\-3\end{bmatrix}$

37. a.
$$\begin{aligned} -3p \qquad\quad - 4r + 6s &= 8 \\ 4p - 2q + 3r \qquad\quad &= -3 \\ 6p + 5q - 2r + 7s &= 1 \end{aligned}$$

b. $p\begin{bmatrix}-3\\4\\6\end{bmatrix} + q\begin{bmatrix}0\\-2\\5\end{bmatrix} + r\begin{bmatrix}-4\\3\\-2\end{bmatrix} + s\begin{bmatrix}6\\0\\7\end{bmatrix} = \begin{bmatrix}8\\-3\\1\end{bmatrix}$

39. F T F F T F T F F T

M1. $[-58, 79, -36, -190]$

M3. Error using + because a and u have different numbers of components.

M5. a. $[-2.4524, 4.4500, -11.3810]$

b. $[-2.45238095238095,$ $4.45000000000000,$ $-11.38095238095238]$

c. $\left[-\frac{103}{42}, \frac{89}{20}, -\frac{293}{21}\right]$

M7. a. $[0.0357, 0.0075, 0.1962]$

b. $[0.03571428571429,$ $0.00750000000000, 0.19619047619048]$

c. $\left[\frac{1}{28}, \frac{3}{400}, \frac{103}{525}\right]$

M9. Error using + because $\mathbf{u}$ is a row vector and $\mathbf{u}^T$ is a column vector.

Section 1.2

1. $\sqrt{26}$ **3.** $\sqrt{26}$ **5.** $\sqrt{478}$

7. $\frac{1}{\sqrt{26}}[-1, 3, 4]$ **9.** -3 **11.** 3

13. $\cos^{-1}\frac{11}{\sqrt{364}} \approx 54.8°$ **15.** $-\frac{43}{3}$

17. $[13, -5, 7]$ or any nonzero multiple of it

19. $15\sqrt{6}$ **21.** -540

25. Perpendicular

27. Parallel with opposite direction

29. Neither

31. The vector $\mathbf{v} - \mathbf{w}$, when translated to start at $(w_1, w_2, \ldots, w_n)$, reaches to the tip of the vector $\mathbf{v}$, so its length is the distance from $(w_1, w_2, \ldots, w_n)$ to $(v_1, v_2, \ldots, v_n)$.

33. $\sqrt{33}$ **35.** 10 **37.** $\frac{100}{\sqrt{3}}$ lb

39. b. $\mathbf{F}_2 = -T_2(\sin\theta_2)\mathbf{i} + T_2(\cos\theta_2)\mathbf{j}$

c. $T_1(\sin\theta_1) - T_2(\sin\theta_2) = 0,\ T_1(\cos\theta_1) + T_2(\cos\theta_2) = 100$

d. $T_1 = \frac{100\sqrt{2}}{\sqrt{3}+1}$ lb, $T_2 = \frac{200}{\sqrt{3}+1}$ lb

M1. $\|\mathbf{a}\| \approx 6.3246$; the two results agree.

M3. $\|\mathbf{u}\| \approx 1.8251$

M5. a. 485.1449

b. Not found by MATLAB

M7. Angle ≈ 0.9499 radians

M9. Angle $\approx 147.4283°$

Section 1.3

1. $\begin{bmatrix}-6 & 3 & 9\\12 & 0 & -3\end{bmatrix}$ **3.** $\begin{bmatrix}2 & 2 & 1\\9 & -1 & 2\end{bmatrix}$

5. $\begin{bmatrix}6 & -3\\-3 & 1\\-2 & 5\end{bmatrix}$ **7.** Impossible

9. $\begin{bmatrix}-130 & 140\\110 & -60\end{bmatrix}$ **11.** Impossible

13. $\begin{bmatrix}27 & -35\\-4 & 26\end{bmatrix}$ **15.** $\begin{bmatrix}20 & -2 & -10\\-2 & 1 & 3\\-10 & 3 & 10\end{bmatrix}$

17. a. $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 128 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

19. $xy = [-14]$, $yx = \begin{bmatrix} -8 & 12 & -4 \\ 2 & -3 & 1 \\ -6 & 9 & -3 \end{bmatrix}$

21. T F F T F T T T F T

23. $(A\mathbf{b})^T(A\mathbf{c}) = \mathbf{b}^T A^T A\mathbf{c}$

27. The (i, j)th entry in $(r + s)A$ is $(r + s)a_{ij} = ra_{ij} + sa_{ij}$, which is the (i, j)th entry in $rA + sA$. Because $(r + s)A$ and $rA + sA$ have the same size, they are equal.

33. Let $A = [a_{ij}]$ be an $m \times n$ matrix, $B = [b_{jk}]$ be an $n \times r$ matrix, and $C = [c_{kq}]$ be an $r \times s$ matrix. Then the ith row of AB is

$$\left[\sum_{j=1}^{n} a_{ij}b_{j1}, \sum_{j=1}^{n} a_{ij}b_{j2}, \ldots, \sum_{j=1}^{n} a_{ij}b_{jr}\right],$$

so the (i, q)th entry in $(AB)C$ is

$$\left(\sum_{j=1}^{n} a_{ij}b_{j1}\right)c_{1q} + \left(\sum_{j=1}^{n} a_{ij}b_{j2}\right)c_{2q} + \cdots + \left(\sum_{j=1}^{n} a_{ij}b_{jr}\right)c_{rq} = \sum_{k=1}^{r}\left(\sum_{j=1}^{n}(a_{ij}b_{jk}c_{kq})\right).$$

Further, the qth column of BC has components

$$\sum_{k=1}^{r} b_{1k}c_{kq}, \sum_{k=1}^{r} b_{2k}c_{kq}, \ldots, \sum_{k=1}^{r} b_{nk}c_{kq},$$

so the (i, q)th entry in $A(BC)$ is

$$a_{i1}\left(\sum_{k=1}^{r} b_{1k}c_{kq}\right) + a_{i2}\left(\sum_{k=1}^{r} b_{2k}c_{kq}\right) + \cdots + a_{in}\left(\sum_{k=1}^{r} b_{nk}c_{kq}\right) = \sum_{j=1}^{n} a_{ij}\left(\sum_{k=1}^{r} b_{jk}c_{kq}\right) = \sum_{j=1}^{n}\left(\sum_{k=1}^{r}(a_{ij}b_{jk}c_{kq})\right) = \sum_{k=1}^{r}\left(\sum_{j=1}^{n}(a_{ij}b_{jk}c_{kq})\right).$$

Because $(AB)C$ and $A(BC)$ are both $m \times s$ matrices with the same (i, q)th entry, they must be equal.

35. a. $n \times m$ **b.** $n \times n$ **c.** $m \times m$

39. Because $(AA^T)^T = (A^T)^T A^T = AA^T$, we see that AA^T is symmetric.

41. a. The jth entry in column vector $A\mathbf{e}_j$ is $[a_{i1}, a_{i2}, \cdots, a_{in}] \cdot \mathbf{e}_j = a_{ij}$. Therefore, $A\mathbf{e}_j$ is the jth column vector of A.

b. (i) We have $A\mathbf{e}_j = \mathbf{0}$ for each $j = 1, 2, \ldots, n$; so, by part a, the jth column of A is the zero vector for each j. That is, $A = O$. (ii) $A\mathbf{x} = B\mathbf{x}$ for each $\mathbf{x}$

if and only if $(A - B)\mathbf{x} = \mathbf{0}$ for each $\mathbf{x}$,

if and only if $A - B = O$ by part (i),

if and only if $A = B$.

45. These matrices do not commute for any value of r.

47. 2989 **49.** 348650

51. 32 **53.** −41558

M1. a. 32 **b.** 14629 **c.** −41558

M3. $\begin{bmatrix} 141 & -30 & -107 \\ -30 & 50 & 18 \\ -107 & 18 & 189 \end{bmatrix}$ **M5.** −117

M7. The expected mean is approximately 0.5. Your experimental values will differ from ours, and so we don't give ours.

Section 1.4

1. a. $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; **b.** $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. a. $\begin{bmatrix} -1 & 1 & 2 & 0 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; **b.** $\begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

5. a. $\begin{bmatrix} 1 & -3 & 0 & 0 & -1 \\ 0 & 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 16 \end{bmatrix}$;

b. $\begin{bmatrix} 1 & -3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

7. $\mathbf{x} = \begin{bmatrix} 5 - 6r \\ 2 - 4r \\ r \end{bmatrix}$, $\begin{bmatrix} -7 \\ -6 \\ 2 \end{bmatrix}$

9. $\mathbf{x} = \begin{bmatrix} 1-2r \\ -2-r-3s \\ r \\ s \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ 3 \\ -2 \end{bmatrix}$ **11.** $\mathbf{x} = \begin{bmatrix} 0 \\ 2 \\ -5 \\ 2 \end{bmatrix}$

13. $x = 2, y = -4$

15. $x = -3, y = 2, z = 4$

17. $\mathbf{x} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ **19.** Inconsistent

21. $\mathbf{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ **23.** $\mathbf{x} = \begin{bmatrix} -8 \\ -23-5s \\ -7+s \\ 2s \end{bmatrix}$

25. Yes **27.** No

29. F F T T F T T T F T

31. $x_1 = -1, x_2 = 3$

33. $x_1 = 2, x_2 = -3$

35. $x_1 = 1, x_2 = -1, x_3 = 1, x_4 = 2$

37. $x_1 = -3, x_2 = 5, x_3 = 2, x_4 = -3$

39. All b_1 and b_2

41. All b_1, b_2, b_3 such that $b_3 + b_2 - b_1 = 0$

43. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **45.** $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

47. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ **49.** $\begin{bmatrix} 1 & -60 & 0 & -15 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -12 & 0 & -3 \end{bmatrix}$

51. $\begin{bmatrix} 2 & -30 & 5 & -10 \\ -4 & 121 & -20 & 40 \\ 0 & -6 & 1 & -2 \\ 0 & 3 & 0 & 1 \end{bmatrix}$

57. $a = 1, b = -2, c = 1, d = 2$

59. $\mathbf{x} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$ **61.** $\mathbf{x} \approx \begin{bmatrix} -1.2857 \\ 3.1429 \\ 1.2857 \end{bmatrix}$

63. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \\ 2 \end{bmatrix}$

65. $\begin{bmatrix} 1 & 2 & 0 & 0 & 3 & -6 \\ 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & -1 & 3 \end{bmatrix}$ **67.** $\mathbf{x} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$

M1. $\mathbf{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ **M3.** Inconsistent

M5. $\mathbf{x} \approx \begin{bmatrix} 1-11s \\ 3-7s \\ s \end{bmatrix}$

M7. $\mathbf{x} = \begin{bmatrix} -13-2r+14s \\ r \\ -5+5s \\ s \end{bmatrix}$

M9. $\mathbf{x} \approx \begin{bmatrix} 0.0345 \\ -0.5370 \\ -1.6240 \\ 0.1222 \\ 0.8188 \end{bmatrix}$

Section 1.5

1. a. $A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

b. The matrix A itself is an elementary matrix.

3. Not invertible

5. a. $A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

b. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(Other answers are possible.)

7. a. $A^{-1} = \begin{bmatrix} -7 & 5 & 3 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \end{bmatrix}$;

b. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{5} \end{bmatrix}$

11. The span of the column vectors is $\mathbb{R}^4$.

13. **a.** $A^{-1} = \begin{bmatrix} -7 & 3 \\ -5 & 2 \end{bmatrix}$ **b.** $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -37 \\ -26 \end{bmatrix}$

15. $x = -7r + 5s + 3t$, $y = 3r - 2s - 2t$, $z = 3r - 2s - t$

17. $\begin{bmatrix} 46 & 33 & 30 \\ 39 & 29 & 26 \\ 99 & 68 & 63 \end{bmatrix}$ **19.** $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & 3 & 4 \end{bmatrix}$.

21. The matrix is invertible for any value of r except $r = 0$.

23. T T T F T T T F F F

25. **a.** No; **b.** Yes

27. **a.** Notice that $A(A^{-1}B) = (AA^{-1})B = IB = B$, so $X = A^{-1}B$ is a solution. To show uniqueness, suppose that $AX = B$. Then $A^{-1}(AX) = A^{-1}B$, $(A^{-1}A)X = A^{-1}B$, $IX = A^{-1}B$, and $X = A^{-1}B$; therefore, this is the only solution.

b. Let $E_1, E_2, \ldots, E_k$ be elementary matrices that reduce $[A \mid B]$ to $[I \mid X]$, and let $C = E_kE_{k-1} \cdots E_2E_1$. Then $CA = I$ and $CB = X$. Thus, $C = A^{-1}$ and so $X = A^{-1}B$.

29. **a.** $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

39. $\begin{bmatrix} 0.355 & -0.0645 & 0.161 \\ -0.129 & 0.387 & 0.0323 \\ -0.0968 & 0.290 & -0.226 \end{bmatrix}$

41. $\begin{bmatrix} 0.0275 & -0.0296 & -0.0269 & 0.0263 \\ 0.168 & -0.0947 & 0.0462 & -0.0757 \\ 0.395 & -0.138 & -0.00769 & -0.0771 \\ -0.0180 & 0.0947 & 0.00385 & 0.0257 \end{bmatrix}$

43. See answer to Exercise 9.

45. See answer to Exercise 41.

47. $\begin{bmatrix} 0.291 & 0.199 & 0.0419 & -0.00828 & -0.272 \\ -0.0564 & 0.159 & 0.148 & -0.0737 & -0.0699 \\ 0.0276 & 0.145 & -0.00841 & -0.0302 & -0.0250 \\ -0.0467 & 0.122 & -0.029 & 0.133 & -0.0084 \\ 0.0116 & -0.128 & -0.0470 & -0.0417 & 0.178 \end{bmatrix}$

M1. 0.001783 **M3.** 0.4397

M5. −418.07 **M7.** −0.001071

Section 1.6

1. A subspace **3.** Not a subspace

5. Not a subspace **7.** Not a subspace

9. A subspace

13. **a.** Every subspace of $\mathbb{R}^2$ is either the origin, a line through the origin, or all of $\mathbb{R}^2$.

b. Every subspace of $\mathbb{R}^3$ is either the origin, a line through the origin, a plane through the origin, or all of $\mathbb{R}^3$.

15. No, because $\mathbf{0} = r\mathbf{0}$ for all $r \in \mathbb{R}$, so $\mathbf{0}$ is not a *unique* linear combination of $\mathbf{0}$.

17. $\{[-1, 3, 0], [-1, 0, 3]\}$

19. $\{[-7, 1, 13, 0], [-6, -1, 0, 13]\}$

21. $\{[-60, 137, 33, 0, 1]\}$ **23.** Not a basis

25. A basis **27.** A basis

29. Not a basis **31.** $\{[-1, -3, 11]\}$

33. $\mathrm{sp}(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k) = \mathrm{sp}(\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m)$ if and only if each $\mathbf{v}_i$ is a linear combination of the $\mathbf{w}_j$ and each $\mathbf{w}_j$ is a linear combination of the $\mathbf{v}_i$.

35. $\mathbf{x} = \begin{bmatrix} -\frac{5}{8} \\ \frac{7}{4} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -3r/4 + s/8 \\ 3r/2 + s/4 \\ r \\ s \end{bmatrix}$

37. $\mathbf{x} = \begin{bmatrix} \frac{5}{3} \\ \frac{5}{3} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -(5r + s)/3 \\ (r + 2s)/3 \\ r \\ s \end{bmatrix}$

39. In this case, a solution is not uniquely determined. The system is viewed as underdetermined because it is insufficient to determine a unique solution.

41. $x - y = 1$
$2x - 2y = 2$
$3x - 3y = 3$

49. A basis **51.** Not a basis

M1. Not a basis **M3.** A basis

M5. The probability is actually 1.

M7. Yes, we obtained the expected result.

Section 1.7

1. Not a transition matrix

3. Not a transition matrix

5. A regular transition matrix

7. A transition matrix, but not regular

9. 0.28 **11.** 0.330 **13.** Not regular

15. Regular **17.** Not regular

19. $\begin{bmatrix} \frac{3}{4} \\ \frac{1}{4} \end{bmatrix}$ **21.** $\begin{bmatrix} \frac{1}{4} \\ \frac{3}{8} \\ \frac{3}{8} \end{bmatrix}$ **23.** $\begin{bmatrix} \frac{12}{35} \\ \frac{8}{35} \\ \frac{15}{35} \end{bmatrix}$

25. T F F T T F T F F T

27. $A^{100} \approx \begin{bmatrix} \frac{12}{35} & \frac{12}{35} & \frac{12}{35} \\ \frac{8}{35} & \frac{8}{35} & \frac{8}{35} \\ \frac{15}{35} & \frac{15}{35} & \frac{15}{35} \end{bmatrix}$ **29.** 0.47 **31.** $\begin{bmatrix} .32 \\ .47 \\ .21 \end{bmatrix}$

33. The Markov chain is regular because T has no zero entry; $\mathbf{s} = \begin{bmatrix} \frac{11}{43} \\ \frac{17}{43} \\ \frac{15}{43} \end{bmatrix}$.

35. $\frac{1}{8}$ **37.** $\begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$

39. The Markov chain is regular because T^2 has no zero entries; $\mathbf{s} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$.

41. $$T = \begin{array}{c} \begin{array}{ccc} \text{D} & \text{H} & \text{R} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \end{bmatrix} \end{array} \begin{array}{l} \text{D} \\ \text{H}. \\ \text{R} \end{array}$$

The recessive state is absorbing, because there is an entry 1 in the row 3, column 3 position.

45. $\begin{bmatrix} \frac{1}{5} \\ \frac{4}{5} \end{bmatrix}$ **47.** $\begin{bmatrix} \frac{5}{9} \\ \frac{4}{9} \end{bmatrix}$ **49.** $\begin{bmatrix} .4037 \\ .3975 \\ .1988 \end{bmatrix}$

51. $\begin{bmatrix} .2115 \\ .3462 \\ .4423 \end{bmatrix}$ **53.** $\begin{bmatrix} .2682 \\ .2235 \\ .1916 \\ .1676 \\ .1490 \end{bmatrix}$

Section 1.8

1. 0001011 0000000 0111001 1111111 1111111 0100101 0000000 0100101 1111111 0111001

3. $x_4 = x_1 + x_2, x_5 = x_1 + x_3, x_6 = x_2 + x_3$

5. Note that each x_i appears in at least two parity-check equations and that, for each combination i,j of positions in a message word, some parity-check equation contains one of x_i, x_j but not the other. As explained following Eq. (2) in the text, this shows that the distance between code words is at least 3. Consequently, any two errors can be detected.

7. **a.** 110 **b.** 001 **c.** 110
d. 001 or 100 or 111 **e.** 101

9. We compute, using binary addition,

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix},$$

where the first matrix is the parity-check matrix and the second matrix has as *columns* the received words.

a. Because the first column of the product is the fourth column of the parity-check matrix, we change the fourth position of the received word 110111 to get the code word 110011 and decode it as 110.

b. Because the second column of the product is the zero vector, the received word 001011 is a code word and we decode it as 001.

c. Because the third column of the product is the third column of the parity-check matrix, we change the third position of the received word 111011 to get the code word 110011 and decode it as 110.

d. Because the fourth column of the product is not the zero vector and not a column of the parity-check matrix, there are at least two errors, and we ask for retransmission.

e. Because the fifth column of the product is the third column of the parity-check matrix, we change the third position of the received word 100101 to get the code word 101101 and decode it as 101.

1. Because we add by components, this follows from the fact that, using binary addition, $1 + 1 = 1 - 1 = 0$, $1 + 0 = 1 - 0 = 1$, $0 + 1 = 0 - 1 = 1$, and $0 + 0 = 0 - 0 = 0$.

3. From the solutions of Exercises 11 and 12, we see that $u - v = u + v$ contains 1's in precisely the positions where the two words differ. The number of places where u and v differ is equal to the distance between them, and the number of 1's in $u - v$ is $\text{wt}(u - v)$, so these numbers are equal.

5. This follows immediately from the fact that $\mathbb{B}^n$ itself is closed under addition modulo 2.

7. Suppose that $d(u, v)$ is minimum in C. By Exercise 13, $d(u, v) = \text{wt}(u - v)$, showing that the minimum weight of nonzero code words is less than or equal to the minimum distance between two of them. On the other hand, if w is a nonzero code word of minimum weight, then $\text{wt}(w)$ is the distance from w to the zero code word, showing the opposite inequality, so we have the equality stated in the exercise.

19. The triangle inequality in Exercise 14 shows that if the distance from received word v to code word u is at most m and the distance from v to code word w is at most m, then $d(u, v) \leq 2m$. Thus, if the distance between code words is at least $2m + 1$, a received word v with at most m incorrect components has a *unique* nearest-neighbor code word. This number $2m + 1$ can't be improved, because if $d(u, v) = 2m$, then a possible received word w at distance m from both u and v can be constructed by having its components agree with those of u and v where the components of u and v agree, and by making m of the $2m$ components of w in which u and v differ opposite from the components of u and the other m components of w opposite from those of v.

21. Let e_i be the word in $\mathbb{B}^n$ with 1 in the ith position and 0's elsewhere. Now e_i is not in C, because the distance from e_i to $000 \ldots 0$ is 1, and $000 \ldots 0 \in C$. Also, $v + e_i \neq w + e_j$ for any two distinct words v and w in C, because otherwise $v - w = e_j - e_i$ would be in C with $\text{wt}(e_j - e_i) = 2$. Let $e_i + C = \{e_i + v \mid v \in C\}$. Note that C and $e_i + C$ have the same number of words. The disjoint sets C and $e_i + C$ for $i = 1, 2, \ldots, k$ contain all words whose distances from some word in C are at most 1. Thus $\mathbb{B}^n$ must be large enough to contain all of these $(n + 1)2^k$ elements. That is, $2^n \geq (n + 1)2^k$. Dividing by 2^k gives the desired result.

23. a. 3 b. 3 c. 4 d. 5 e. 6 f. 7

25. $x_9 = x_1 + x_2 + x_3 + x_6 + x_7$, $x_{10} = x_5 + x_6 + x_7 + x_8$, $x_{11} = x_2 + x_3 + x_4 + x_6 + x_8$, $x_{12} = x_1 + x_3 + x_4 + x_5$ (Other answers are possible.)