Section 2.1 Independence and Dimension

28. T F T F T T F T T T T T F T

33. Suppose that

$$r_1[1,0,1] + r_2[2,s,3] + r_3[1,-s,0] = [0,0,0]$$

, so that

$$[r_1 + 2r_2 + r_3, sr_2 - sr_3, r_1 + 3r_2] = [0,0,0].$$

We solve that linear system

$$\begin{aligned} r_1 + \; & 2r_2 & +r_3 & = 0, \\ & sr_2 & -sr_3 & = 0, \\ r_1 + \; & 3r_2 & & = 0. \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & s & -s & 0 \\ 1 & 3 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & s & -s & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

We see that the vectors are dependent for all values $s \in \mathbb{R}$.

34. Suppose scalar $r, s \in \mathbb{R}$ such that $rA\vec{v} + sA\vec{w} = \vec{0}$. Then $A(r\vec{v} + s\vec{w}) = \vec{0}$.

Since A is invertible, we have

$$A^{-1}A(r\vec{v} + s\vec{w}) = A^{-1}\vec{0}$$

Thus

$$r\vec{v} + s\vec{w}) = \vec{0}$$

Therefore, $r = s = 0$ since $\vec{v}$ and $\vec{w}$ are linearly independent. Hence, $A\vec{v}$ and $A\vec{w}$ are linearly independent.

36. Suppose scalar $r, s \in \mathbb{R}$ such that $r\vec{v} + s\vec{w} = \vec{0}$. Then

$$\vec{0} = A\vec{0} = A(r\vec{v} + s\vec{w}) = rA\vec{v} + sA\vec{w}.$$

Since $A\vec{v}$ and $A\vec{w}$ are linearly independent, we have $r = s = 0$. Thus $\vec{v}$ and $\vec{w}$ are linearly independent.