Section 2.2 the rank of a matrix

12. Let A be an $n \times n$ matrix. Then

$$null(A) = n - rank(A) = n - rank(A^T) = null(A^T).$$

14. Let A be $m \times n$ matrix. Every vector in the column space of AC is of the form $\vec{v} = (AC)\vec{x}$ for some $\vec{x} \in \mathbb{R}^n$. For every $\vec{x}$, $(C\vec{x}) \in \mathbb{R}^n$. Then $\vec{v} = A(C\vec{x})$ which is the vector in the column space of A. Thus $colspace(AC) \subseteq colspace(A)$.

18. By exercise 14, we have $colspace(AC) \subseteq colspace(A)$. Thus

$$\dim(colspace(AC)) \leq \dim(colspace(A))$$

That is, $rank(AC) \leq rank(A)$

20. Yes! Apply exercise 14 as follows:

$$rank(AC) = rank((AC)^T) = rank(C^T A^T) \leq rank(C^T) = rank(C)$$