Section 3.1

6. Define $(f \oplus g) = \max\{f(x), g(x)\}$, for all $x \in \mathbb{R}$ and $(rf)(x) = rf(x)$, for all $x \in \mathbb{R}$. Assume $z(x)$ is the $\vec{0}$, that is for all $f(x)$, $z(x) = f(x) \oplus (-f)(x) = \max\{f(x), (-f)(x)\} = \max\{f(x), -f(x)\}$.
Let $f(x) = 1, z(x) = f(x) \oplus (-f)(x) = \max\{1, -1\} = 1$. However, by **A3**, $z(x) \oplus (-f)(x) = (-f)(x) = -1 \neq \max\{1, -1\}$. Therefore, $\vec{0}$ does not exists.

16. The set P_n of all polynomials in x, with real coefficients and of degree less or equal to n, together with zero polynomial. Noticed that the set P of all polynomials in x with real coefficients is a vector space. (Example 2 in textbook 3-1) Since P_n is a subset of P. P_n is a vector space if $\vec{0} \in P_n$ and P_n is closed under vector addition and scalar multiplication.
Let $p(x) = p_n x^n + ... + p_1 x + p_0$, $q(x) = q_n x^n + ... + q_1 x + q_0$ are two polynomials of degree $\leq n$ and let r is a real number.
Then

$$\begin{aligned} (rp)(x) &= rp_n x^n + ... + rp_1 x + rp_0 \\ (p+q)(x) &= (p_n + q_n)x^n + ... + (p_1 + q_1)x + (p_0 + q_0) \end{aligned}$$

are polynomials of degree $\leq n$. Hence, the set P_n is closed under vector addition and scalar multiplication.

18. (a) Matrix multiplication is a vector space operation on the set $M_{m\times n}$ of $m \times n$ matrices.
False. Vector space operations are just scalar multiplication and vector addition.

(b) Matrix multiplication is a vector space operation on the set $M_{n\times n}$ of square $n \times n$ matrices.
False. Vector space operations are just scalar multiplication and vector addition.

(c) Multiplication of any vector by the zero scalar always yields the zero vector.
True.

(d) Multiplication of a non-zero vector by a non-zero scalar always yields a non-zero vector.
True.

(e) No vector is its own additive inverse.
False. The zero vector $\vec{0}$ is its own additive inverse.

(f) The zero vector is the only vector that is its own additive inverse.
True.

(g) Multiplication of two scalars is of no concern to the definition of a vector space.
False. Check **S3**.

(h) Every vector spaces has at least two vectors.
False. $\{\vec{0}\}$ with normal vector addition and scalar multiplication is a vector space.

(i) Every vector space has at least one vector.
True. Every vector space contains a zero vector.