Section 3.2

2. The set is NOT a subspace of P since it is not closed under vector addition. For example,$p(x) = x^4 + x^3$ and $q(x) = -x^4$ are both in the set, but $p(x) + q(x) = x^3$ is not in the set.

4. $W = \{f|f(1) = 0\}$ is a subspace of F. You should verify that W contains zero vector, and closed under vector addition, and closed under scalar multiplication, which is proved below.
$z(x) = 0$ is the zero vector in F. $z(x) \in W$ since $g(1) = 0$.
Suppose f, g are functions satisfying $f(1) = g(1) = 0$ and r is a real number.

$$\begin{aligned} (rf)(1) &= rf(1) = 0 \\ (f+g)(1) &= f(1) + g(1) = 0 + 0 = 0 \end{aligned}$$

8. Note that

$$1 = 1(1 + 2x) + (-2)x$$

and

$$x = 0(1 + 2x) + 1(x)$$

,so $sp(1, x)$ is contained in $sp(1 + 2x, x)$. Next,

$$1 + 2x = 1(1) + 2(x)$$

and

$$x = 0(1) + 1(x),$$

so $sp(1+2x, x)$ is contained in $sp(1, x)$. Thus we conclude that $sp(1, x) = sp(1+2x, x)$.

12. The set of vectors is dependent. Supposer

$$1 + r_2(4x + 3) + r_3(3x–4) + r4(x^2 + 2) + r_5(x–x^2) = 0.$$

Then

$$(r_4–r_5)x^2 + (4r_2 + 3r_3 + r_5)x + (r_1 + r_2–4r_3 + 2r_4) = 0.$$

Thus we solve the system $\begin{cases} r_4 - r_5 = 0 \\ 4r_2 + 3r_3 + r_5 = 0 \\ r_1 + 3r_2 - 4r_3 + 2r_4 = 0 \end{cases}$.

We row reduce the augmented matrix

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & -4 & 2 & 0 & 0 \end{array}\right] \simeq \left[\begin{array}{ccccc|c} 1 & 0 & -25/4 & 0 & 5/4 & 0 \\ 0 & 1 & 3/4 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

Since the third and fifth columns do not contain a pivot, r_3 and r_5 are free variables, so we can easily find a non-trivial solution for r_1, r_2, r_3, r_4, r_5. Thus the set is dependent.

20. $(x-1)^2 = (x^2+1) + (-2)x$, so the set of vectors is dependent and hence is NOT a basis for P_2.

26. TTFTFTTFTT