Section 5-2 Diagonalization

17. Prove that, for every square matrix A all of whose eigenvalues are real, the product of its eigenvalues is $\det(A)$

Answer: If the characteristic polynomial of A is $p(\lambda) = |A - \lambda I|$, then $p(0) = |A| = \det(A)$. Also,

$$p(\lambda) = (-1)^n (\lambda - \lambda_1)(\lambda - \lambda_2) \cdots (\lambda - \lambda_n)$$

, so

$$p(0) = (-1)^{2n} \lambda_1 \lambda_2 \cdots \lambda_n = \lambda_1 \lambda_2 \cdots \lambda_n = \det(A).$$