

$$\int a^m dx = \frac{a^m}{\ln a} + C$$

$$\int e^m dx = e^m + C$$

Math

$$\int \frac{du}{u^2 - a^2}$$

古爾丁定理

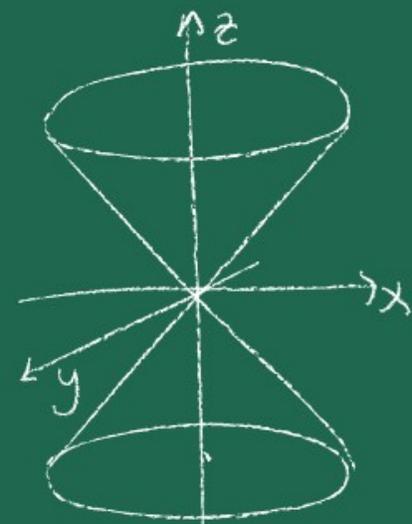
$$y = f(x) \pm f'(x)$$

$$y = f(x) \pm g(x) \pm f'(x) \pm g'(x)$$

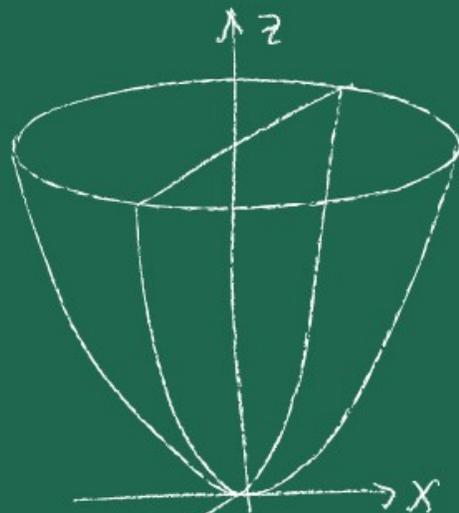
$$y = kf(x) = kf'(x)$$

$$\int \frac{du}{u} = \ln|u|$$

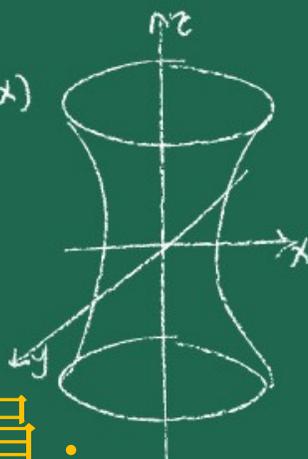
$$y = f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$



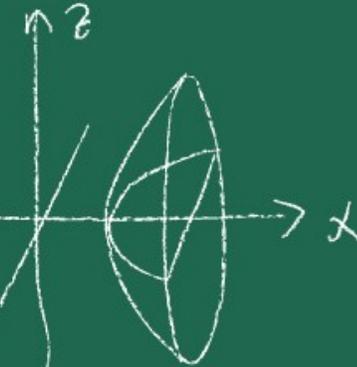
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$$



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 - 劉康昱 410931111



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\int a^m dx = \frac{a^m}{\ln a} + c$$

Pappus-Guldinus theorem

(帕普斯-古爾丁定理)

Math

$$\int \frac{du}{u^2 - a^2}$$

一、定義

$$y = f(x) = f'(x)$$

$$y = f(x) \pm g(x) = f'(x) \pm g'(x)$$

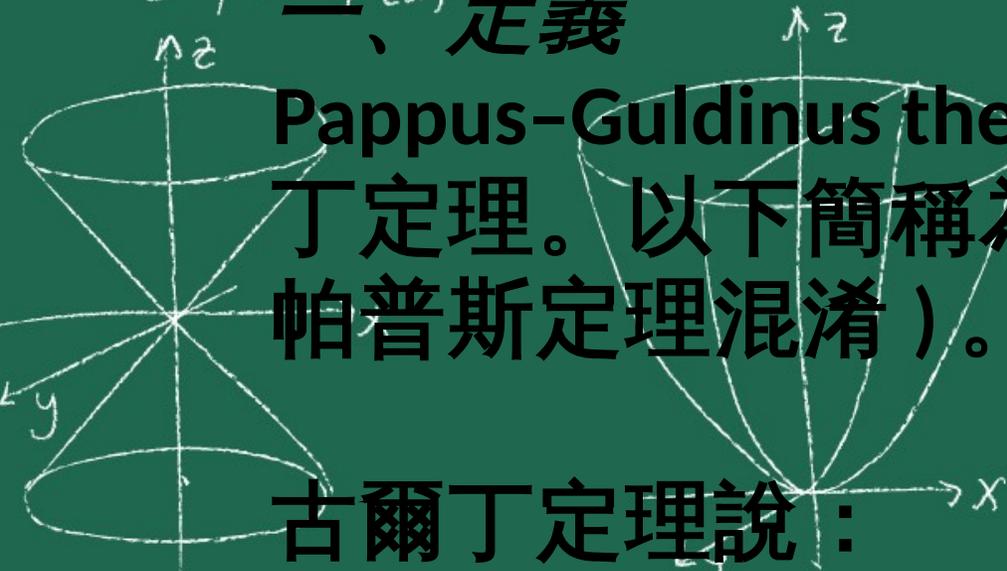
$$y = kf(x) = kf'(x)$$

$$\int \frac{du}{u} = \ln |u|$$

Pappus-Guldinus theorem, 中文譯作帕普斯-古爾丁定理。以下簡稱為古爾丁定理 (為避免和幾何的帕普斯定理混淆)。

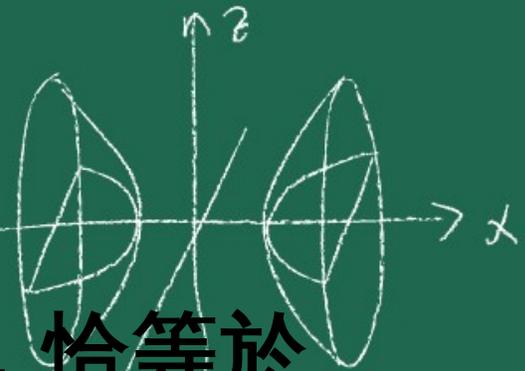
古爾丁定理說：

一個平面圖形繞著軸旋轉出的旋轉體體積，恰等於此圖形面積乘以此圖形質心所走路徑長。



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int \sin u du = -\cos u + c$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + c$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

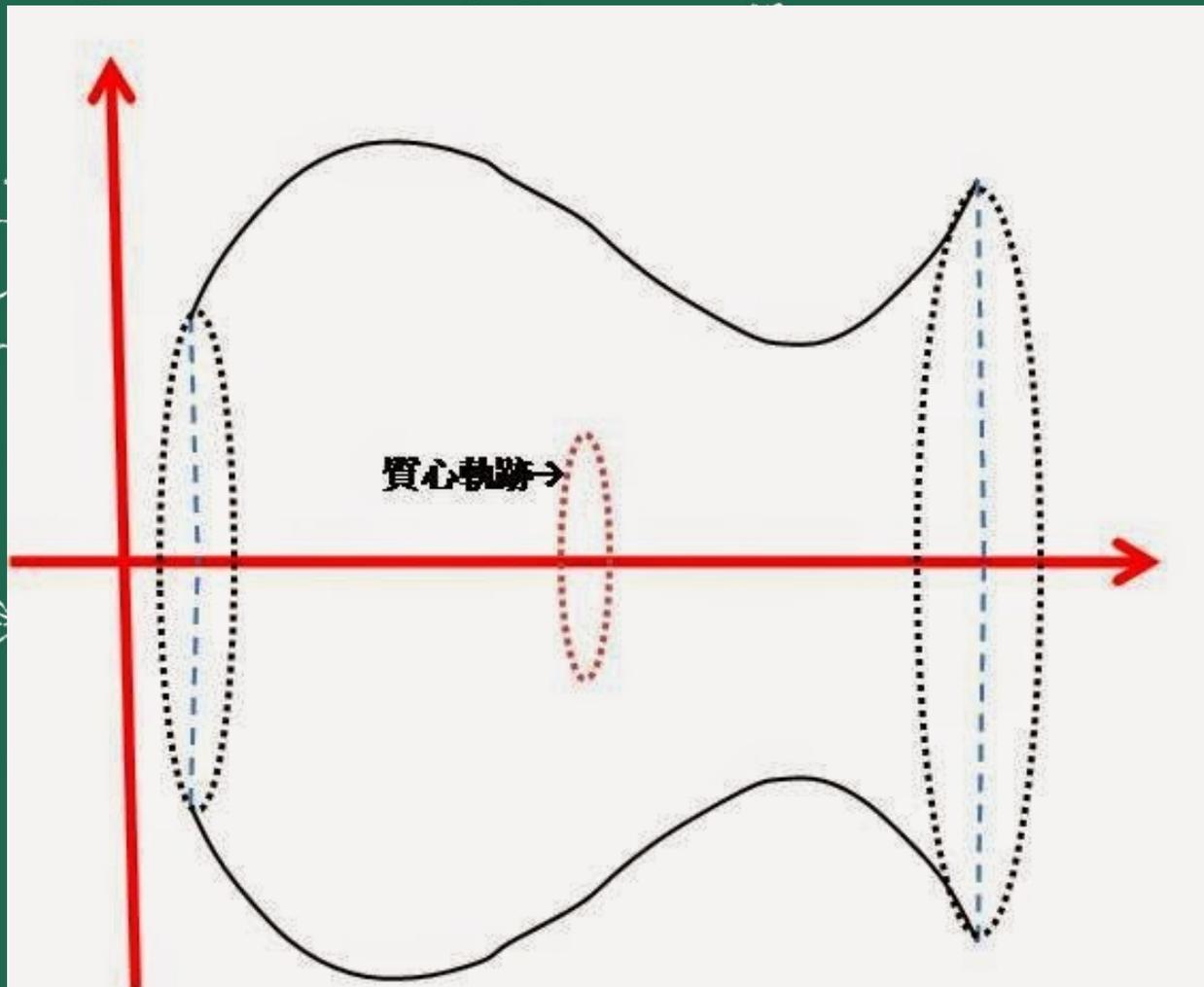
$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

旋轉示意圖:

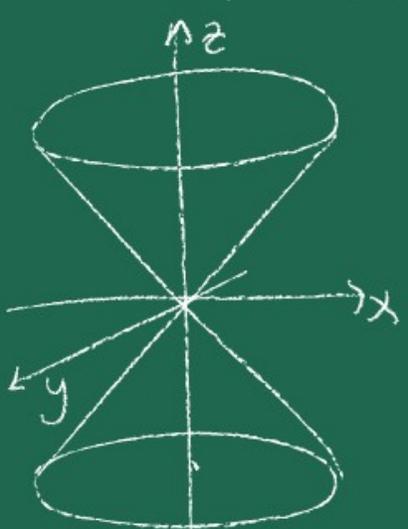
$$\int e^m du = e^m + c$$

Math

$$\int \frac{du}{u^2 - a^2}$$



$$y = f(x) = f'(x)$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

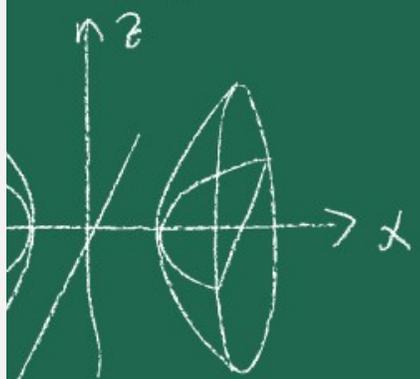


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$f(x) = kf'(x)$$

$$\int \frac{du}{u} = \ln|u|$$

$$f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + c$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

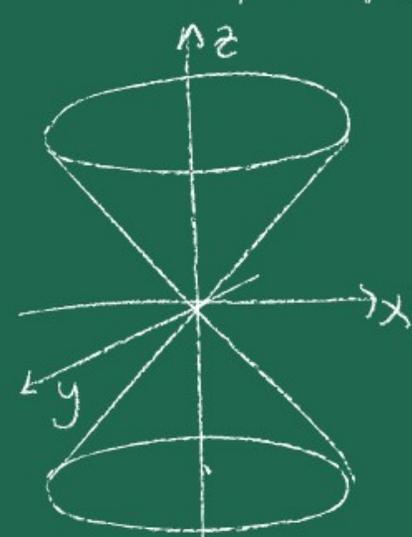
$$\int u^m du = \frac{u^{m+1}}{m+1} + C \quad \text{證明}$$

$$\int e^m dx = e^m + C$$

Math

$$\int \frac{du}{u^2 - a^2}$$

$$y = f(x) = f'(x)$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int u^n du = \frac{u^{n+1}}{n+1}$$

先備定理：

1. $y = f(x), a \leq x \leq b$, 這段曲線以及 $x = a, x = b, x$ 軸為成一個圖形
此圖形繞 x 軸旋轉的旋轉體體積：

$$V = \int_a^b \pi [f(x)]^2 dx$$

2. 平面圖形重心座標：

$$(x_{CM}, y_{CM}) = \left(\frac{\iint x dA}{\iint dA}, \frac{\iint y dA}{\iint dA} \right)$$

3. $y = f(x), a \leq x \leq b$, 與 x 軸圍出面積

$$A = \int_a^b f(x) dx$$

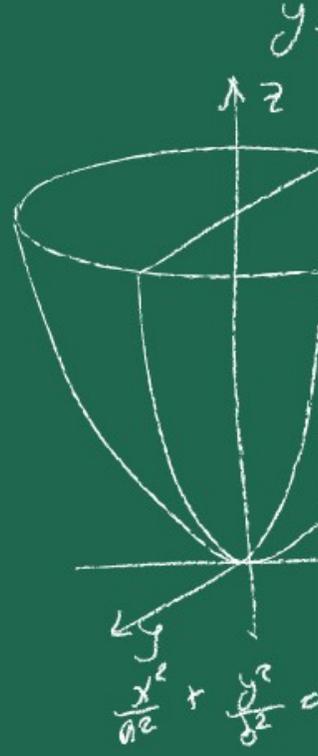
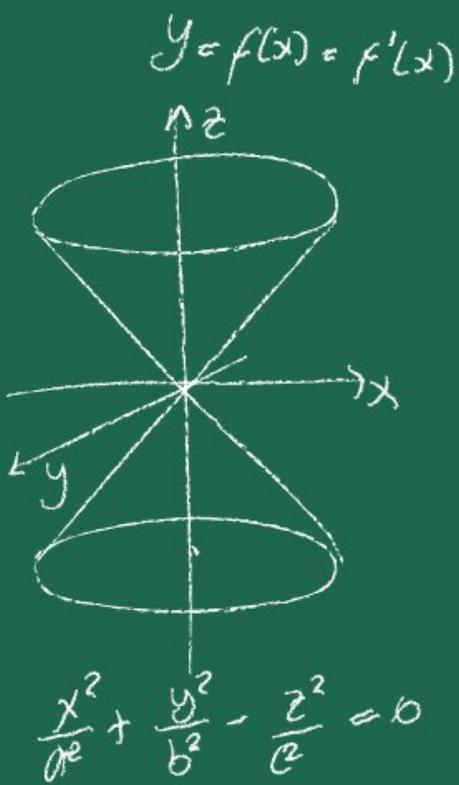
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\int \frac{dx}{x^2} = -\frac{1}{x} + C$$

正式開始證明：

質心路徑長

$$\begin{aligned}
 &= 2\pi \cdot y_{cm} \\
 &= 2\pi \cdot \frac{\int_a^b \int_0^{f(x)} y dy dx}{\int_a^b f(x) dx} \\
 &= 2\pi \cdot \frac{\int_a^b \frac{1}{2} y^2 \Big|_{y=0}^{f(x)} dx}{\int_a^b f(x) dx} \\
 &= \pi \cdot \frac{\int_a^b [f(x)]^2 dx}{\int_a^b f(x) dx}
 \end{aligned}$$



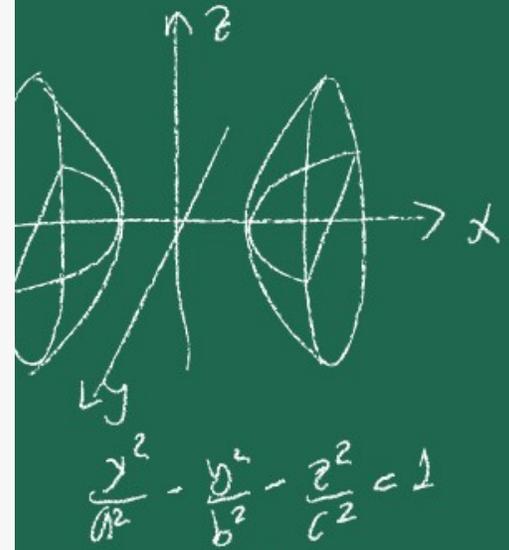
$$\int M^n du = \frac{M^{n+1}}{n+1} + c$$

$$\int \frac{du}{u^2 - a^2}$$

$kf(x) = kf'(x)$

$$\int \frac{du}{u} = \ln|u|$$

$y = f(x)g(x) = f'(x)g(x) + f(x)g'$

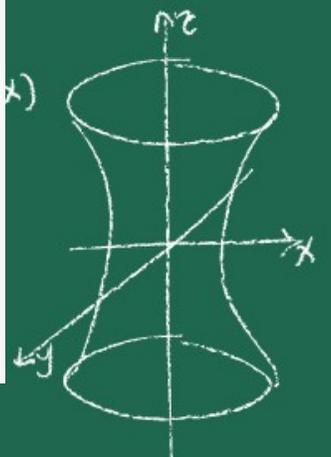


$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Math

圖形面積

$$= \int_a^b f(x) dx$$



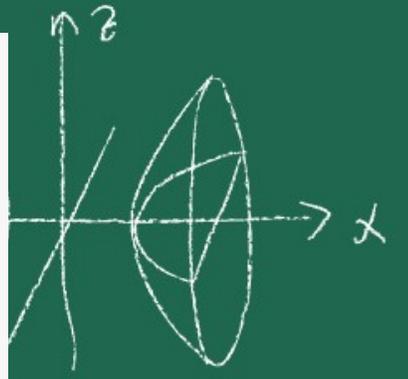
$$y = kf(x) = kf'(x)$$

$$\int \frac{du}{u} = \ln|u|$$

$$y = f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

旋轉體體積

$$= \int_a^b \pi \cdot [f(x)]^2 dx$$

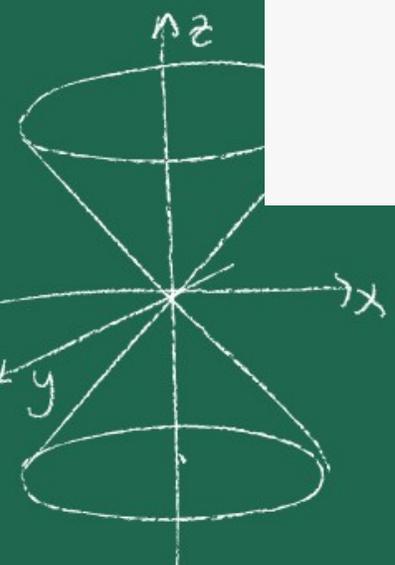


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

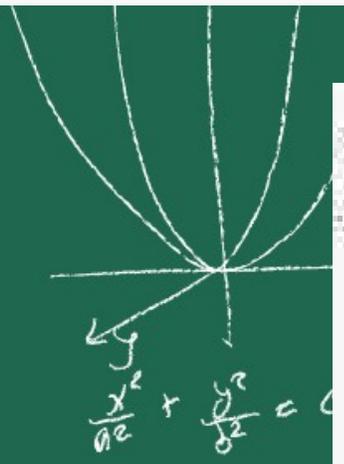
$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int a^m dx = \frac{a^m x}{m} + c$$

$$y = f(x)$$

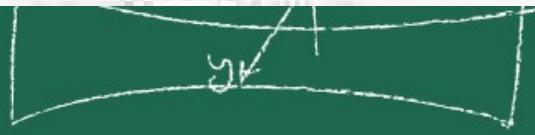


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + c$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

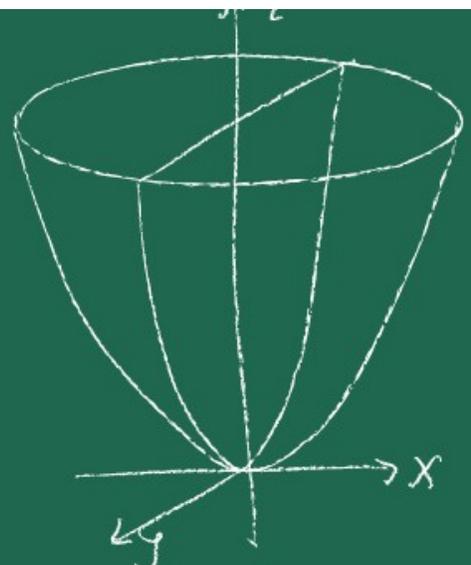
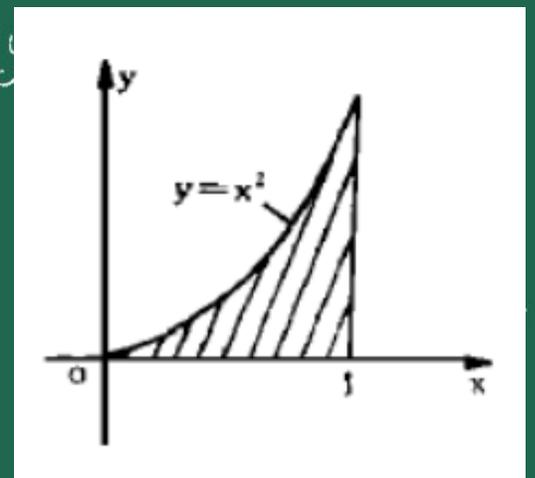
三、題目講解

Math

$$\int e^m du = e^m + c$$

$$\int \frac{du}{u^2 - a^2}$$

(1) $y = x^2$ 和 x 轴、 $x = 1$ 所围图形, 绕 y 轴

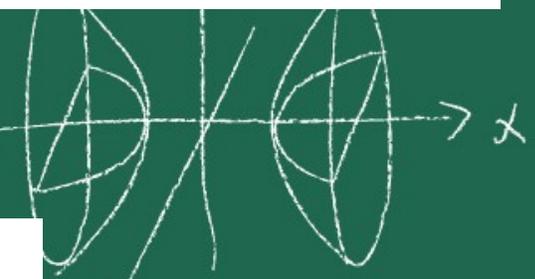


解:

$$V = \int_0^1 2\pi x \cdot x^2 dx = 2\pi \int_0^1 x^3 dx = 2\pi \left. \frac{x^4}{4} \right|_0^1 = \frac{\pi}{2}$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + c$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

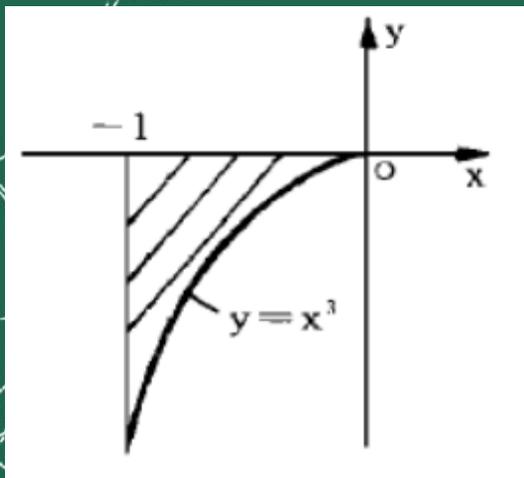
$$\int a^m dx = \frac{a^m}{\ln a} + c$$

$$\int e^m dx = e^m + c$$

Math

$$\int \frac{du}{u^2 - a^2}$$

(2) $y = x^3$ 和 x 轴, $x = -1$ 所围图形, 绕 y 轴

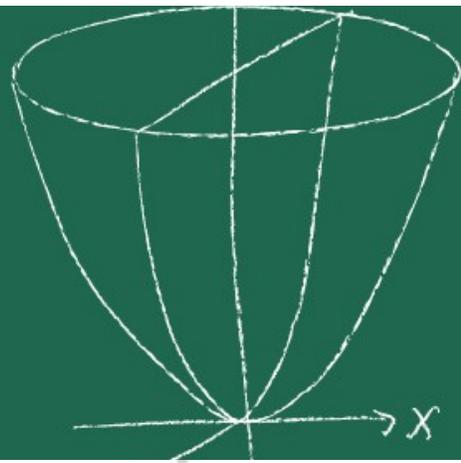
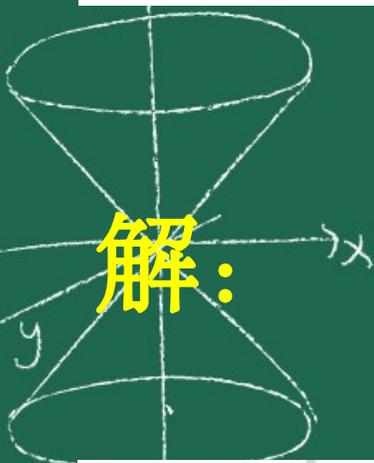


$$f(x) = k f'(x)$$

$$\int \frac{du}{u} = \ln |u|$$

$$f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

解:



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int \sec u du = -\cos u + c$$

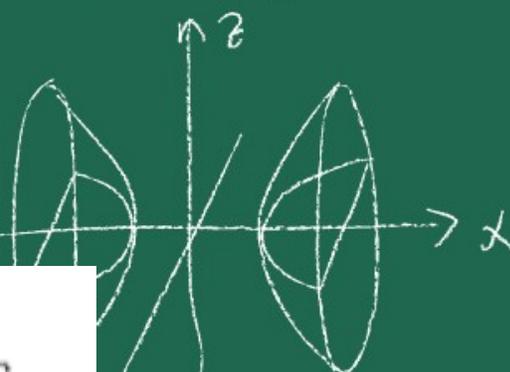
$$V = \int_{-1}^0 2\pi |x| |x^3| dx = \int_{-1}^0 2\pi (-x)(-x^3) dx = \int_{-1}^0 2\pi x^4 dx = 2\pi \frac{x^5}{5} \Big|_{-1}^0 = \frac{2}{5}\pi$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + c$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

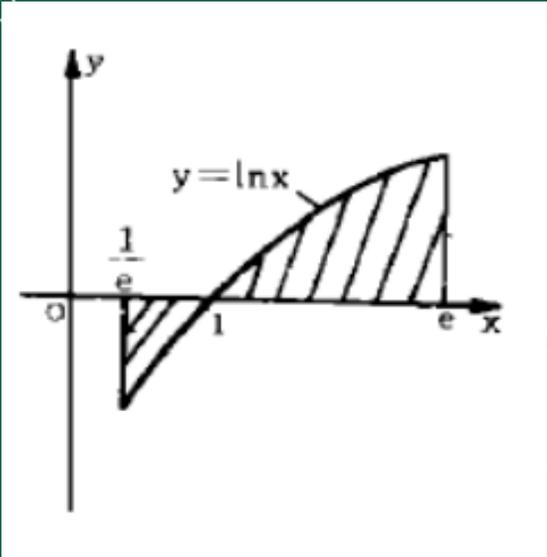


$$\int a^m dx = \frac{a^m}{\ln a} + C$$

$$\int e^m dx = e^m$$

Math

$x_1 = e^{-1}, x_2 = e$ 和 x 轴所围图形绕 y 轴



解:

$$V = \int_{e^{-1}}^e 2^c x |\ln x| dx = \int_{e^{-1}}^1 2^c x (-\ln x) dx + \int_1^e 2^c x \ln x dx$$

$$= -2^c \left(\frac{x^2}{2} \ln x - \frac{x^2}{4} \right) \Big|_{e^{-1}}^1 + 2^c \left(\frac{x^2}{2} \ln x - \frac{x^2}{4} \right) \Big|_1^e = c \left(1 + \frac{e^2}{2} - \frac{3}{2} e^{-1} \right)$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

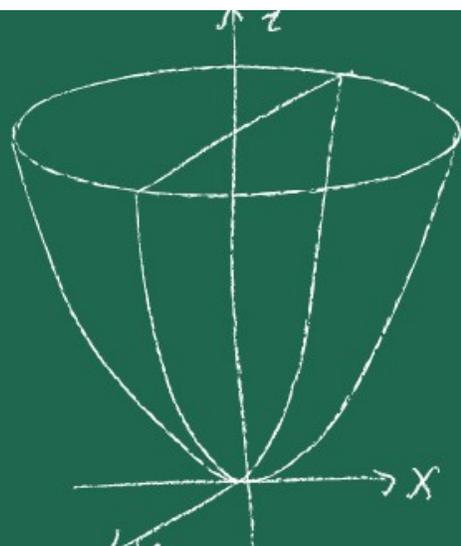
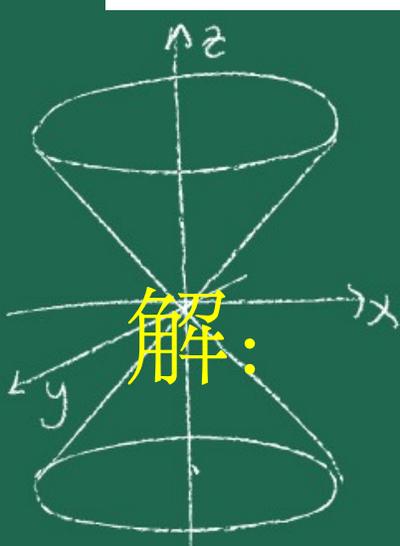
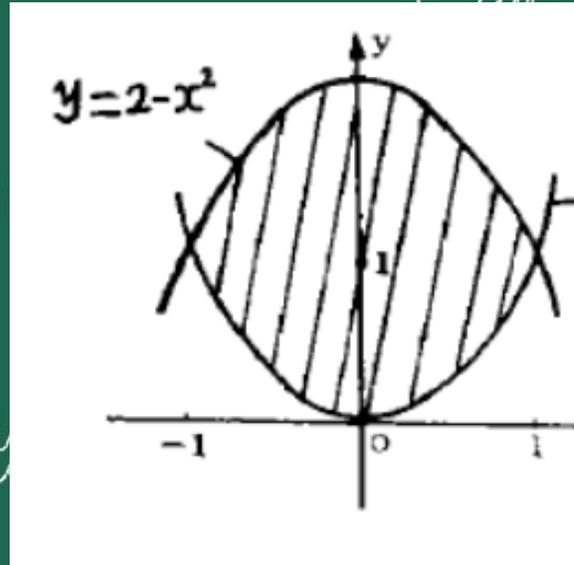
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int a^m dx = \frac{a^m}{\ln a} + C$$

$$\int e^m dx = e^m + C$$

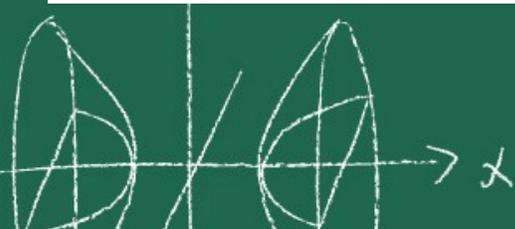
Math

(4) $y = x^2$ 和 $y = 2 - x^2$ 所围图形, 绕 x 轴.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

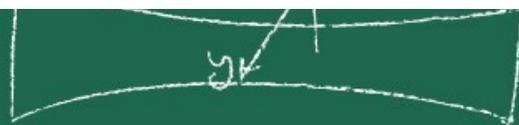
$$\int \sin \mu dx = -\cos \mu + C$$



解:

$$V = 2 \int_0^1 2^c x [(2 - x^2) - x^2] dx = 4^c \int_0^1 x (2 - 2x^2) dx = 4^c (x^2 - \frac{1}{2} x^4) \Big|_0^1$$

$$\int u^n dx = \frac{u^{n+1}}{n+1} + C$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

說例 9
漂亮題

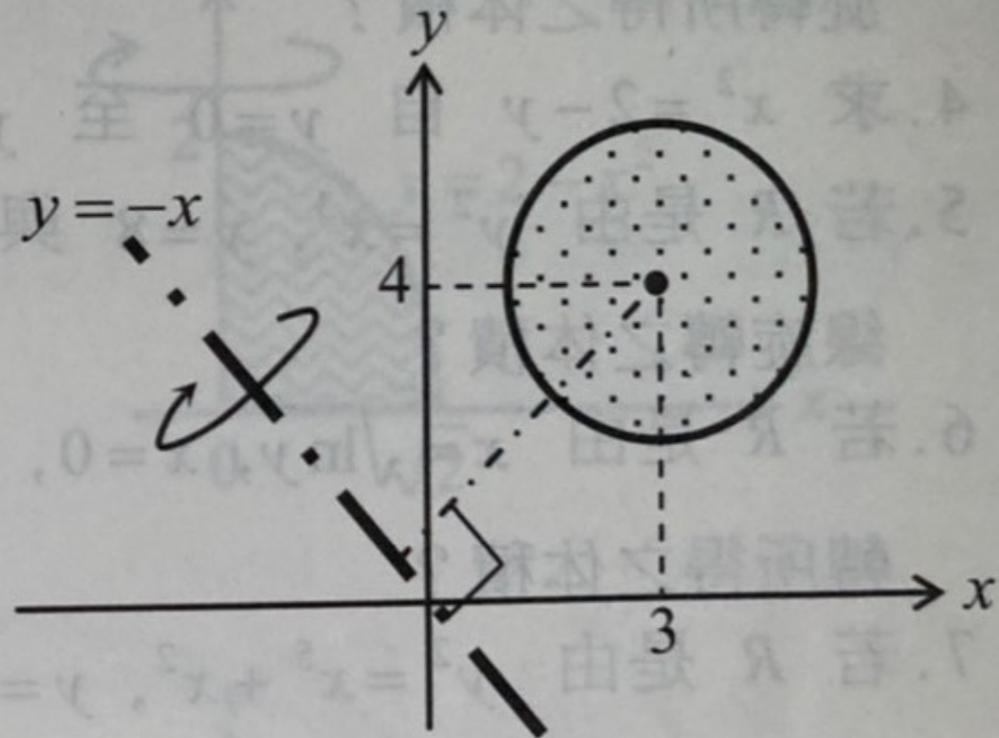
求圓 $(x-3)^2 + (y-4)^2 = 4$ 繞直線 $y = -x$ 一圈所得之旋轉體
體積？ (中興轉)

[解] 先求圓心 $(3, 4)$ 到 $y = -x$ 之距離

$$\text{得 } d = \frac{|3+4|}{\sqrt{1+1}} = \frac{7}{\sqrt{2}}$$

利用 Pappus 定理得

$$V = (\pi \cdot 2^2) \cdot \left(2\pi \cdot \frac{7}{\sqrt{2}} \right) = 28\sqrt{2}\pi^2。$$



$$n+1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C$$

$$\int a^m dx = \frac{a^m}{\ln a} + c$$

$$\int e^m dx = e^m + c$$

Math

$$\int \frac{dx}{a^2 - x^2}$$

參考資料 :

$$y = f(x) \pm g(x) = f'(x) \pm g'(x)$$

$$y = kf(x) = kf'(x)$$

$$\int \frac{dx}{x} = \ln|x|$$

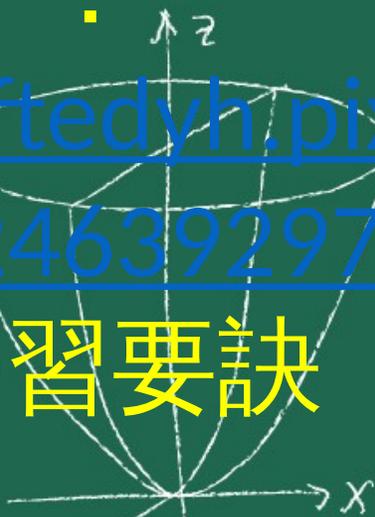
<https://giftedyh.pixnet.net/blog/post/375633428>

<http://m24639297.blogspot.tw/>

微積分學習要訣



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

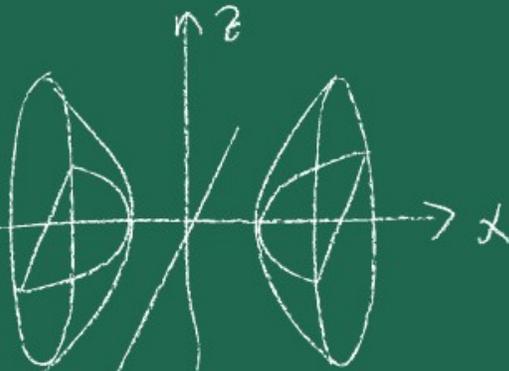


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$$

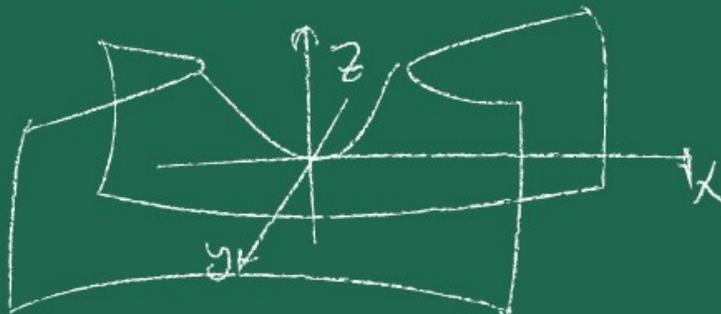


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

$$\int \sin mx dx = -\cos mx + c$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$