# 第四組

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# 第一題

Let $f(n, k)$ be the number of ways of distributing $k$ candies to $n$ children so that each child receives at most 2 candies. For example, if n=3, then $f(3,7) = 0$, f (3, 6) = 1 and f(3, 4) = 6.

令 $f(n, k)$ 表示為 $n$ 與 $k$ 之關係。將 $k$ 個糖果分配給 $n$ 個小朋友，每個小朋友最多拿到兩顆糖果。舉例，假設 n=3，則 f (3, 7) =0， f (3, 6) = 1 以及 f (3, 4) =6。

請判斷以下式子之值

$$f(2006,1) + f(2006,4) + f(2006,7) + \cdots + f(2006,1000) + f(2006,1003)$$

# 第二題

Let ABC be an acute-angled triangle. Inscribe a rectangle DEFG in this triangle so that D is on AB, E is on AC and both F and G are on BC. Describe the locus of (i.e., the curve occupied by) the intersections of the diagonals of all possible rectangles DEFG.

三角形ABC是一個銳角三角形，讓長方形DEFG在三角形ABC中，D在AB線上，E在AC線上，F和G在BC線上。請描述 所有可能的長方形DEFG的對角線的交點 的軌跡。

# 第三題

In a rectangular array of nonnegative real numbers with m rows and n columns, each row and each column contains at least one positive element. Moreover, if a row and a column intersect in a positive element, then the sums of their elements are the same. Prove that m = n

在mxn的矩陣中，每列每行至少包含一個正數，此外，如果一列跟一行的交界是正數，則它們的元素和會相等，證明m = n

# 第四題

Consider a round-robin tournament with 2n + 1 teams, where each team plays each other team exactly once. We say that three teams X, Y and Z, form a cycle triplet if X beats Y , Y beats Z, and Z beats X. There are no ties.

(a) Determine the minimum number of cycle triplets possible.
(b) Determine the maximum number of cycle triplets possible.

有一場循環賽，有2n+1組隊伍參加，每組隊伍只會對上其他隊伍一次。假設有三組隊伍x，y，z ，如果x贏y，y贏z，z贏x，這稱為一個三連環，且比賽沒有平手的情況發生。

(a) 最少有可能有幾組三連環
(b) 最多有可能有幾組三連環

# 第五題

The vertices of a right triangle ABC inscribed in a circle divide the circumference into three arcs. The right angle is at A, so that the opposite arc BC is a semicircle while arc AB and arc AC are supplementary. To each of the three arcs, we draw a tangent such that its point of tangency is the midpoint of that portion of the tangent intercepted by the extended lines AB and AC. More precisely, the point D on arc BC is the midpoint of the segment joining the points D0 and D00 where the tangent at D intersects the extended lines AB and AC. Similarly for E on arc AC and F on arc AB. Prove that triangle DEF is equilateral.

圓內切直角三角形ABC的頂點將圓周分成三個弧線
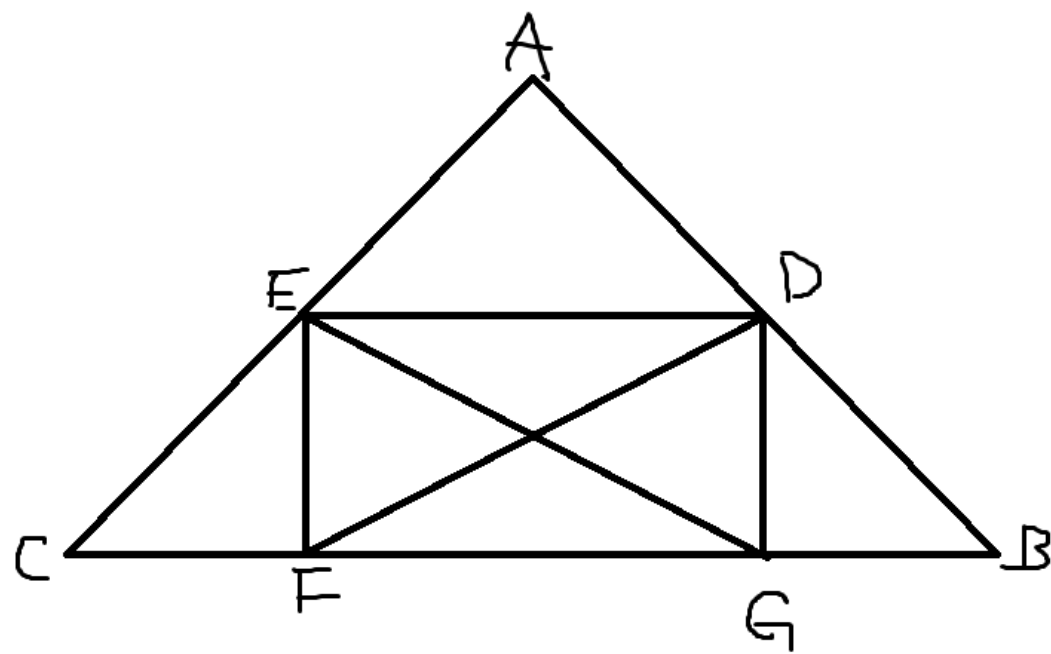直角是A 這樣相對的圓弧BC是半圓 而圓弧AB和圓弧AC是互補
在這三個弧線上 畫切線 使它的切點為被延長線擷取的部分切線的中點
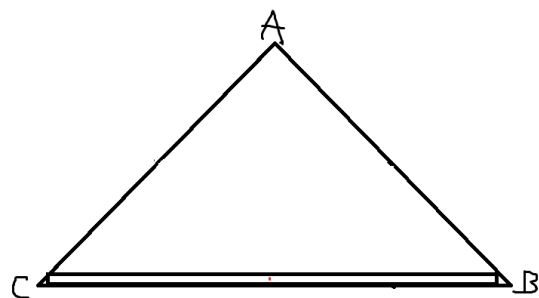更確切的說 圓弧BC上點D是D'跟D''的中點 該點的交點處的切線與延長線相交 同理可知 E是圓弧AC的中點 F是圓弧AB的中點
證明三角形DEF是等邊的

簡單圖示

考慮兩種極端狀況，長方形DEFG的對角線交點必落在三角形中線，且會逼近線段CB跟三角形ABC中點