第六組

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1. ABCD is a convex quadrilateral for which AB is the longest side. Points M and N are located on sides AB and BC respectively, so that each of the segments AN and CM divides the quadrilateral into two parts of equal area. Prove that the segment MN bisects the diagonal BD.

ABCD四邊形，AB是最長邊，M點N點分別在AB BC 邊上，AN CM線段分四邊形為兩相等面積，證明M N 線段平分對角線BD

2. Determine all functions f defined on the set of rational numbers that take rational values for which
$f(2f(x) + f(y)) = 2x + y$, for each x and y

對於每個x和y，確定在一有理數集合上定義的所有函數f，這些函數採用有理值，其中$f(2f(x) + f(y)) = 2x + y$

3. Let a, b, c be positive real numbers for which $a+b+c=1$. Prove that

$$\frac{a-bc}{a+bc}+\frac{b-ca}{b+ca}+\frac{c-ab}{c+ab}\le\frac{3}{2}.$$

設a,b,c為正實數，符合a+b+c=1

證明

$$\frac{a-bc}{a+bc}+\frac{b-ca}{b+ca}+\frac{c-ab}{c+ab}\le\frac{3}{2}.$$

4. Determine all functions f defined on the natural numbers that take values among the natural numbers for which

$$(f(n))^p \equiv n \mod f(p)$$

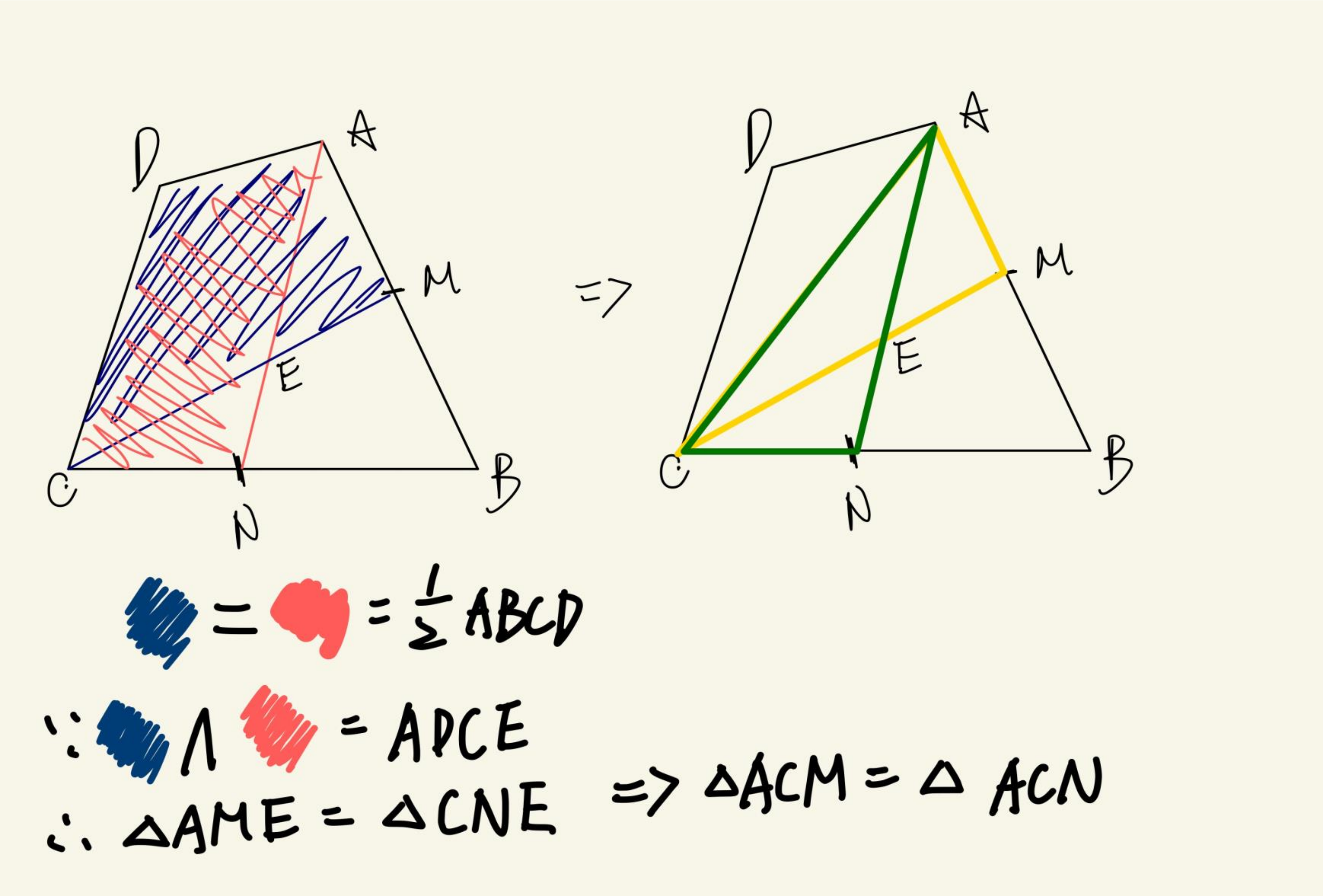
for all $n \in \mathbf{N}$ and all prime numbers p.

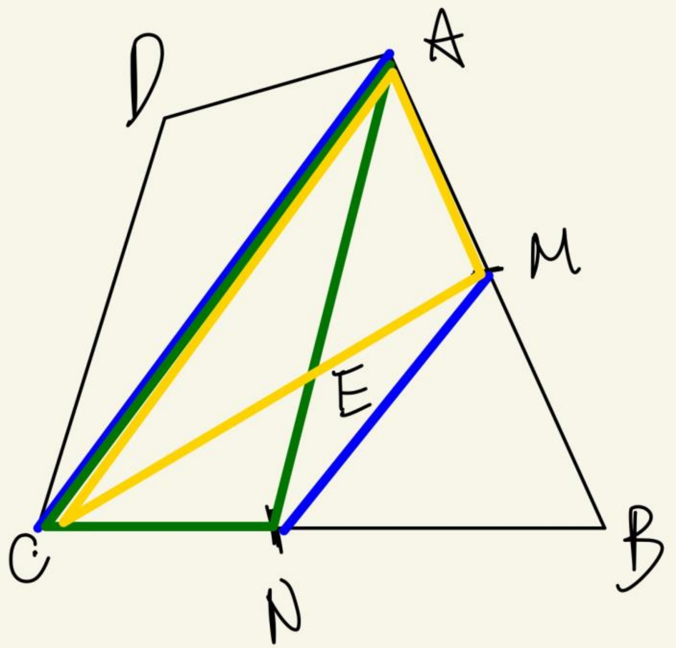
找到所以由自然數定義的函數f,該數取自於(f(n))^p≡n mod f(p),n屬於自然數,p為所有質數

5. A self-avoiding rook walk on a chessboard (a rectangular grid of unit squares) is a path traced by a sequence of moves parallel to an edge of the board from one unit square to another, such that each begins where the previous move ended and such that no move ever crosses a square that has previously been crossed, i.e., the rook's path is non-self-intersecting. Let R(m, n) be the number of self-avoiding rook walks on an m × n (m rows, n columns) chessboard which begin at the lower-left corner and end at the upper-left corner. For example, R(m, 1) = 1 for all natural numbers m; R(2, 2) = 2; R(3, 2) = 4; R(3, 3) = 11. Find a formula for R(3, n) for each natural number n.

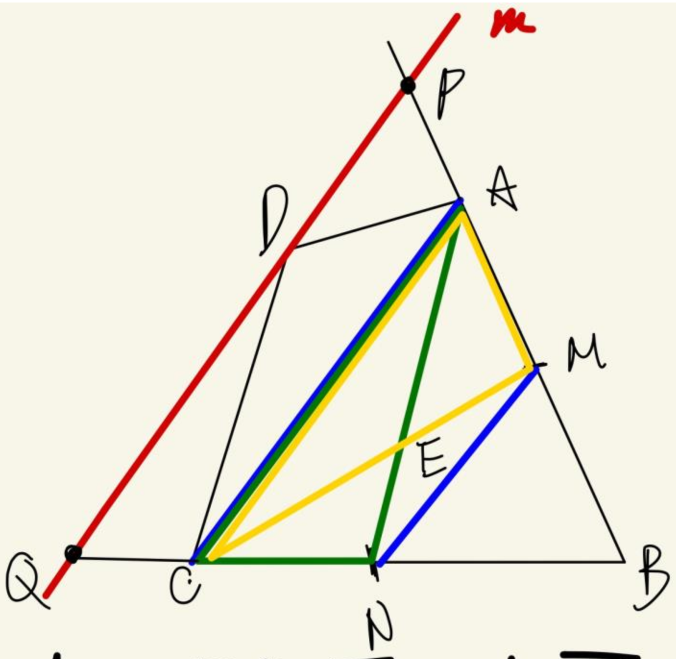
國際象棋中有個棋子叫“車”，棋盤上“車”的自避行走是指“車”這樣行走的一條踪跡路徑：從一個方格出發穿過兩個方格之間的公共邊界（不能斜著走）進入另一個方格，但走過的方格不能再走。即“車”的路徑是不自交的。令R(m，n)表示m×n的棋盤（m行，n列）上自避行走的“車”從左下角走到左上角的路徑的數目。例如：R(m，1)＝1，R(2，2)＝2，R(3，2)＝4，R(3，3)＝11。求出R(3，n)的表達式（用n表示）。

第一題





$\Rightarrow \overline{AC} \parallel \overline{MN}$



Let $m \parallel \overline{AC}$ and $\overline{MN}$

$\Rightarrow \triangle MPC = \triangle MAC + \triangle CAP$

$= \triangle MAC + \triangle CAD$

$= MADC = \triangle BMC$

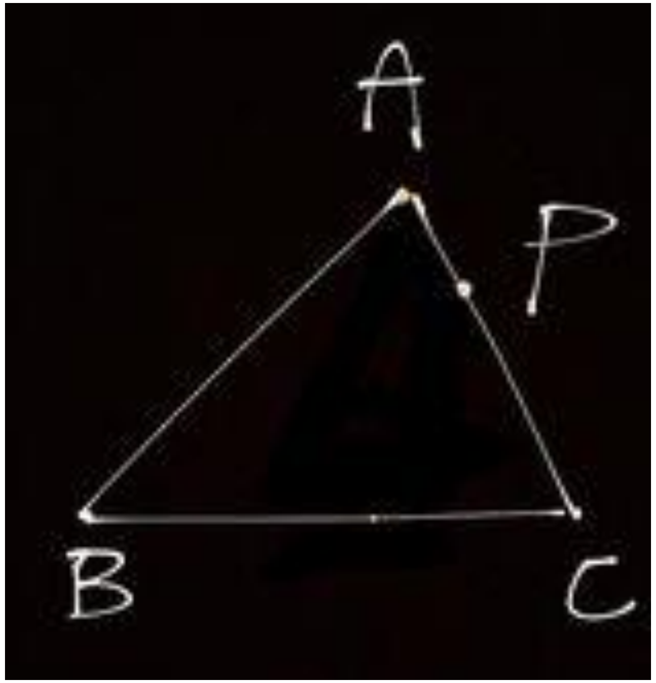
$\because \triangle BMC = \triangle CMP = \frac{1}{2} ABCD$

$\therefore \overline{BM} = \overline{MP} \Rightarrow \overline{BN} = \overline{NQ}$

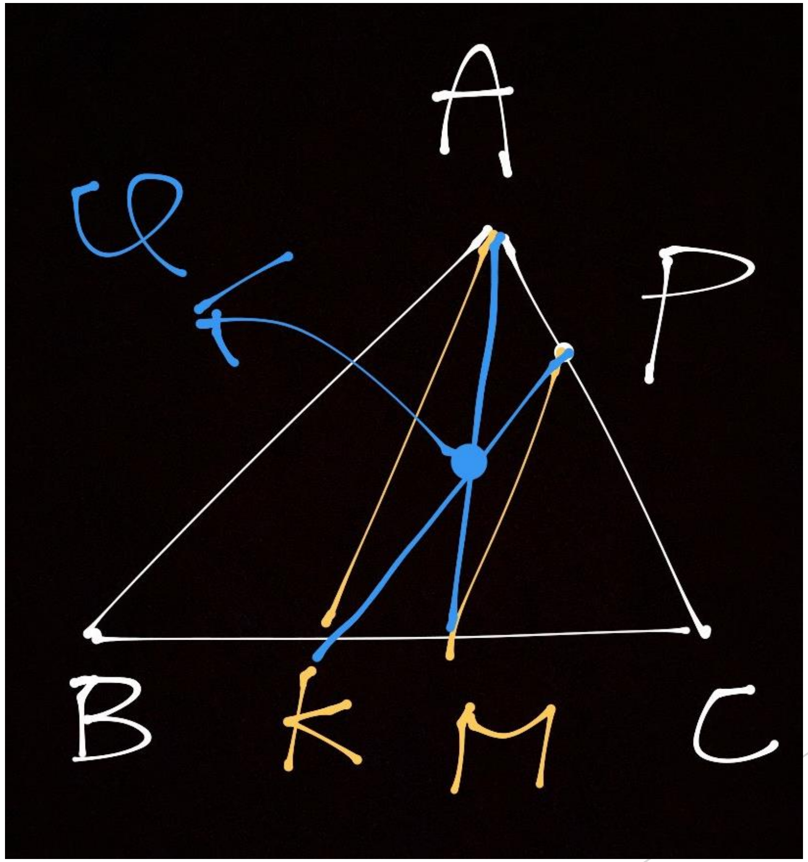
$\Rightarrow \overline{MN} : \overline{PQ} = 1:2 \Rightarrow$ 平分 $\overline{BD}$

相似題

過一定點P作一直線平分三角形ABC面積，P為AC邊上任一點。



作法：
1.作BC邊邊的中線AM，並連PM線段。
2.過A作AK//PM。
3.連直線PK即為所求。



證明：

AK//PM，故⊿APQ＝⊿QMK，AM是中線，故⊿AMC＝⊿AMB

⊿CPK＝四邊形CPQM+⊿KMQ

＝四邊形CPQM+⊿AKQP

＝⊿AMC

＝(1/2)⊿ABC，所以直線PK平分⊿ABC