第六組 2017

組員：410931227 廖柔雅
410931223 高林雅星
410831212 王承瀚
410831110 藍立翔
410831108 黃暐傑

1. Let a, b, and c be non-negative real numbers, no two of which are equal. Prove that

$$a^2 /(b - c)^2 + b^2 /(c - a)^2 + c^2 /(a - b)^2 > 2$$

1. a,b,c 為三個非負實數,每兩個互不相等，證明

$$a^2 /(b - c)^2 + b^2 /(c - a)^2 + c^2 /(a - b)^2 > 2$$

2. Let f be a function from the set of positive integers to itself such that, for every n, the number of positive integer divisors of n is equal to f(f(n)). For example, f(f(6)) = 4 and f(f(25)) = 3. Prove that if p is prime then f(p) is also prime.

(代數)

2.設f為一個在正整數集合內的方程式 符合對所有正整數n n的正因數=f(f(n)) 例如 f(f(6)) = 4 和 f(f(25)) = 3。證明如果p是質數 則f(p)也是質數

3. Let n be a positive integer, and define Sn = {1, 2, . . . , n}. Consider a non-empty subset T of Sn. We say that T is balanced if the median of T is equal to the average of T. For example, for n = 9, each of the subsets {7}, {2, 5}, {2, 3, 4}, {5, 6, 8, 9}, and {1, 4, 5, 7, 8} is balanced; however, the subsets {2, 4, 5} and {1, 2, 3, 5} are not balanced. For each n ≥ 1, prove that the number of balanced subsets of Sn is odd.

(數論)

3.令n為正整數，Sn={1,2,.......n}。考慮到從Sn裡找非零集合T，當T的中位數等於T的平均數時，我們稱它是平衡的。舉例來說：n=9，{7},{2,5},{2,3,4},{5,6,8,9},{1,4,5,7,8}這些集合均平衡的，但是{2,4,5},{1,2,3,5}並不平衡。對於所有n≥1，證明Sn裡所有平衡集合的個數為偶數。

4. Points P and Q lie inside parallelogram ABCD and are such that triangles ABP and BCQ are equilateral. Prove that the line through P perpendicular to DP and the line through Q perpendicular to DQ meet on the altitude from B in triangle ABC.

(幾何)

4. PQ在平行四邊形ABCD中，且 ABP、BCQ為等邊三角形。求過P點且與DP垂直的線與過Q點與DQ垂直的線，在三角形ABC中B點上的高有交點。

5. One hundred circles of radius one are positioned in the plane so that the area of any triangle formed by the centres of three of these circles is at most 2017. Prove that there is a line intersecting at least three of these circles.

(幾何)

5. 將三個半徑為100的圓放置在平面上，使得三個圓的中心連線形成任意三角形的面積最多為2017。證明有一條線與這些圓中的至少三個相交。

1.

Let a, b, and c be non-negative real numbers, no two of which are equal. Prove that

$$\frac{a^2}{(b-c)^2}+\frac{b^2}{(c-a)^2}+\frac{c^2}{(a-b)^2}>2$$

$\Rightarrow a>b>c\geq 0$

∵左式和 >0

∴每項最小值相加 >2 時

其值必 >2

Let a, b, and c be non-negative real numbers, no two of which are equal. Prove that

$$\frac{a^2}{(b-c)^2}+\frac{b^2}{(c-a)^2}+\frac{c^2}{(a-b)^2}>2$$

min: $(b-c)^2$ 的最大值

$\Rightarrow c=0$

min: $(c-a)^2$ 的最大值

$\Rightarrow c=0$

$$\frac{a^2}{(b-c)^2} + \frac{b^2}{(c-a)^2} + \frac{c^2}{(a-b)^2} > \mathbf{2}$$

, $c=0$

$$\Rightarrow \frac{a^2}{b^2} + \frac{b^2}{a^2} \geq 2$$

$$\Rightarrow \frac{a^4+b^4}{a^2b^2} > 2$$

算幾不等式: $\frac{a^4+b^4}{2} \geq \sqrt{a^4b^4} = a^2b^2$

$$\Rightarrow \frac{a^4+b^4}{2} \geq a^2b^2 \Rightarrow \frac{a^4+b^4}{a^2b^2} \geq 2$$

$$\frac{a^4+b^4}{a^2b^2} \geq 2$$

但 $a \neq b$

所以 $a^4 \neq b^4$ ，"="不成立

⇒ 得證 $$\frac{a^4+b^4}{a^2b^2} > 2$$

$$\therefore \frac{a^2}{(b-c)^2} + \frac{b^2}{(c-a)^2} + \frac{c^2}{(a-b)^2} > 2$$