數學解題方法第9組

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1.

Let S be a set of n ≥ 3 positive real numbers. Show that the largest possible number of distinct integer powers of three that can be written as the sum of three distinct elements of S is n − 2.

今有一集合S之元素為所有包含n之正實數且n大於等於3

試證若以三為底數之三的n-2次方可被表示為S之某三不同數之總和

第一題解法

若今有x為S之最大數我們x與其他兩數之相加總和必會在x到3x之間由題義可知3的n-3次方也可以被S/{x}表示。

延伸:把所有3改成5那要證明之數應為n-4

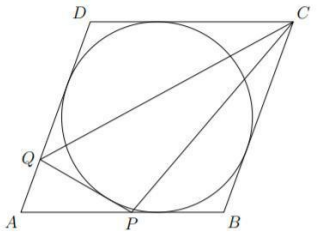
2.

A circle is inscribed in a rhombus ABCD. Points P and Q vary on line segments AB and AD, respectively, so that P Q is tangent to the circle. Show that for all such line segments P Q, the area of triangle CP Q is constant.

菱形ABCD中有一個圓圈。P點和Q點在線段AB上變化

和AD分別使PQ與圓相切。顯示所有這樣的行

在段PQ處，三角形CPQ的面積是恆定的。



3.

A purse contains a finite number of coins, each with distinct positive integer values. Is it possible that there are exactly 2020 ways to use coins from the purse to make the value 2020?

錢包包含有限數量的硬幣，每個硬幣都有不同的正整數值。是否有可能有2020種方法湊成2020?

4.

Let S = {1, 4, 8, 9, 16, . . .} be the set of perfect powers of integers, i.e. numbers of the form n k where n, k are positive integers and k ≥ 2. Write S = {a1, a2, a3 . . .} with terms in increasing order, so that a1 < a2 < a3 · · · . Prove that there exist infinitely many integers m such that 9999 divides the difference am+1 − am.

集合S={1 4 8 9 16……}為的集合，且a、皆為正整數。今將S由小到大排列，試證存在有限個正整數m使的可以被9999整除。

5.

There are 19, 998 people on a social media platform, where any pair of them may or may not be friends. For any group of 9, 999 people, there are at least 9, 999 pairs of them that are friends. What is the least number of friendships, that is, the least number of pairs of people that are friends, that must be among the 19, 998 people?

有19998個人在某個社群網路上，其中的任兩人都可能互為朋友關係。現在隨意取9999人，皆會出現最少9999對朋友。請問整個群體中，最少有幾對朋友?