第一組

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題目

- 1. Each of ten boys has 100 pokemon cards. In each move, one of the boys gives one card to each of the other boys. What is the minimal number of moves before each boy has a different number of cards?

- 1.十個男孩中每個人都有100 張神奇寶貝卡。在每一步中,其中一個男孩給其他每個人一張卡片。在每個男孩有不同數量的卡牌之前,最少的移動次數是多少?

- 2. There are 64 positive integers in the squares of an 8×8 board. Whenever the board is covered by 32 dominoes, the sum of the two integers covered by each domino is unique. Is it possible that the largest integer on the board does not exceed 32?

- 2. 8×8 棋盤的方格中有 64 個正整數。每當棋盤被 32 張多米諾骨牌覆蓋時,且每張多米諾骨牌所覆蓋的兩個整數之和是唯一的。有沒有可能棋盤上的最大整數不超過32?

- 3. The circumcentre O of a quadrilateral ABCD does not lie on either diagonal. The circumcircle of AOC passes through the midpoint of BD. Prove that the circumcircle of BOD passes through the midpoint of AC.

- 3. 四邊形 ABCD 的外心 O 不在對角線上。 AOC 的外接圓經過 BD 的中點。證明BOD的外接圓經過AC的中點。

- 4. The 2016 pairwise sums of 64 numbers are recorded on one piece of paper. They are distinct and positive. The 2016 pairwise products of the same 64 numbers are recorded on another piece of paper. They are also distinct and positive. Later, it is forgotten which piece is which. Is it still possible to determine which piece is which?

- 4. 2016對和為64 的數字被記錄在一張紙上。它們是相異正數。 2016 對積一樣為 64 個數字的被記錄在另一張紙上。它們也是相異正數。後來忘記了是哪一塊。是否仍然可以確定哪一塊是哪一塊？

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- 5. Is it possible to cut a 1 × 1 square into two pieces which can cover a disk of diameter greater than 1?

- 5. 是否能將 1 × 1 的正方形切成兩塊，以覆蓋直徑大於 1 的圓盤？

- 6. Alice chooses a polynomial P(x) with integer coefficients. In each move, Bob gives Alice an integer a, and Alice tells him the number of different integer solutions of the equation P(x) = a. Bob may not give Alice the same number twice. Determine the minimal number of moves for Bob to make Alice tell him a number that she has told him before, regardless of the polynomial chosen by Alice.

- 6. Alice 選了一個整係數多項式 P(x)。在每一步中，Bob 給 Alice 一個整數 a，Alice 告訴他 P(x) = a 的不同整數解的數量。 Bob 不能給 Alice 相同的數兩次。不管Alice 選擇的多項式為何，確定一下Bob 必讓 Alice 告訴他一個她之前講過數字的最小移動步數。

- 7. A finite number of frogs are placed on distinct integer points on the real line. At each move, a single frog jumps by 1 to the right provided that the new location is unoccupied. Altogether, the frogs make n moves, and this can be done in m ways. Prove that if they jump by 1 to the left instead of to right, they can still make n moves in m ways.

- 7. 有限數量的青蛙被放置在實數線上的不同整數點上。每次移動時，如果新位置是空的，一隻青蛙會向右跳 1。青蛙總共移動 n 次，這可以用 m 種方式完成。證明如果他們向左而不是向右跳躍 1，他們仍然可以以 m 種方式進行 n 次移動。

第一題

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- 1.十個男孩中每個人都有100 張神奇寶貝卡。在每一步中，其中一個男孩給其他每個人一張卡片。在每個男孩有不同數量的卡牌之前，最少的移動次數是多少？

- 解法：1. 霍華德-哈利姆的解決方案

假設經過n次移動，每個男孩都有不同數量的小精靈卡片。 對於$1 < k < 10$，讓第k個男孩在m_k步中送出卡片，在$n-m_k$步中收到卡片。 那麼他最終將擁有$100 + (n - m_k) - 9 m_k = 100 + n - 10 m_k$卡片。 每個男孩最終都會得到不同數量的牌，當且僅當我的牌是不同的。 現在$n = m_1 + m_2 + \ldots + m_{10}$. 由於每個$m_k$是一個非負整數，n的最小值是$0 + 1 + 2 + \ldots + 9 = 45$。 - - --

類似題

英文

Each of ten boys has 1000 pokemon cards. In each move, one of the boys gives three card to each of the other boys. What is the minimal number of moves before each boy has a different even number of cards?

中文

- 十個男孩中每個人都有
1000 張神奇寶貝卡。
在每一步中，其中一個
男孩給其他每個人3張卡
片。在每個男孩有不同
偶數的卡牌之前，最少
的移動次數是多少？

解法(英文)

Suppose after n moves, every boy has a different enen number of Pokemon cards. For $1 \le k \le 10$, let the kth boy give away cards in m_k moves and receives cards in $n - m_k$ moves. Then he will end up with $1000 + 3(n - m_k) - 27m_k = 1000 + 3n - 30m_k$ cards. Each boy will end up with a different even number of cards if and only if m_k are distinct. Now $n = m_1 + m_2 + \cdots + m_{10}$. Since each m_k is a non-negative integer, and $1000 - 30m_k$ is even, n must be even ,the minimum value of n is $(0 + 1 + 2 + \cdots + 8)$ +10 = 46.

解法(中文)

假設n步移動後，每個男孩會有不同偶數的神奇寶貝卡，

對$1 \leq k \leq 10$，第k位男孩m_k步分送卡片，$n-m_k$步獲得卡片，

- 那麼，男孩最後會有$1000 + 3(n - m_k) - 27m_k = 1000 + 3n - 30m_k$張卡片。

- 每個男孩結束有不同偶數張卡，若且唯若m_k不同。現在n等於m_1加到m_{10}，因為m_k是非負整數，$1000-30m_k$必是偶數，n則必須也是偶數，最小值就是0加到8再加10即46。