第一組

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翻譯

- 1. A triangle has an angle of measure θ. It is dissected into several triangles. Is it possible that all angles of the resulting triangles are less than θ, if
- (a) $\theta = 70^\circ$;
- (b) $\theta = 80^\circ$?
- 1. 三角形的量角為 θ。 它被分解成幾個三角形。 得到的三角形是否有可能所有角度都小於 θ
- (a) $\theta = 70^\circ$； (b) $\theta = 80^\circ$？

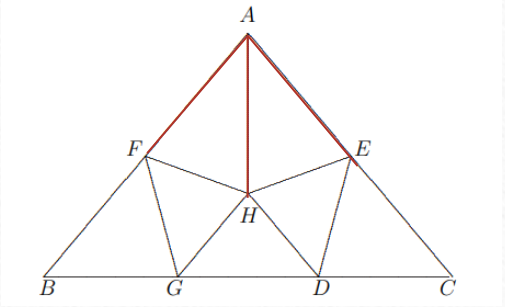
(a) $\theta = 70^\circ$

(a) Solution by Noble Zhai.

Suppose the task is possible. In the resulting triangulation, the 70° angle must be subdivided into at least two angles. Hence one of these angles is at most 35°. In the triangle to which it belongs, one of the other two angles is at least $1/2\ (180^\circ - 35^\circ) = 72.5^\circ$. This is a contradiction.

假設上述成立。在生成的三角剖分中，必須將 70° 角細分為至少兩個角。因此，這些角度之一至多為 35°。在它所屬的三角形中，其他兩個角之一至少為 1/2 (180◦ -35◦) = 72.5◦ 。則矛盾。

(b) $\theta = 80^\circ$



In the diagram below, ABC is a triangle with AB = AC and ∠CAB = 80◦. It is dissected into seven triangles where AF = AH = AE, HF = HG = HD = HE, HG is parallel to AB and HD is parallel to AC. Then ∠DHG = 80◦, ∠HGD = ∠GDH = 50◦, ∠HAE = ∠HAF = 40◦, ∠AFH = ∠AHF = ∠FHG = ∠DHE = ∠EHA = ∠AEH = 70◦ and ∠BGF = ∠CDE = 75◦. The other angles all have measure 55◦. If we move D and G a little closer to each other, we can make all angles to have measure less than 80◦.

在上圖中，三角形ABC 中 AB = AC 且 ∠CAB = 80。它分為七個三角形，其中 AF = AH = AE，HF = HG = HD = HE，HG 平行於 AB，HD 平行於 AC。那麼 ∠DHG = 80°, ∠HGD = ∠GDH = 50°, ∠HAE = ∠HAF = 40°, ∠AFH = ∠AHF = ∠FHG = ∠DHE = ∠EHA = ∠AEH = 70° 和 ∠BGF = ∠CDE = 75°。其他角度都有 55° 。如果我們將 D 和 G 彼此靠近一點，我們可以使所有角度都小於 80°。

2. Alice and Brian are playing a game on the real line. To start the game, Alice places a checker on a number x where $0 < x < 1$. In each move, Brian chooses a positive number d. Alice must move the checker to either $x + d$ or $x - d$. If it lands on 0 or 1, Brian wins. Otherwise the game proceeds to the next move. For which values of x does Brian have a strategy which allows him to win the game in a finite number of moves?

2. Alice和Brian在實線上玩遊戲。 開始遊戲時，Alice 將棋子放在數字 x 上，其中 $0 < x < 1$。在每一步中，Brian 選擇一個正數 d。 Alice 必須將棋子移動到 $x + d$ 或 $x - d$。 如果它落在 0 或 1 上，Brian 獲勝。 否則遊戲進行下一步。 對於哪個 x 值，Brian 可以在有限步數內必勝？

3. A polynomial $x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-2}x^2 + a_{n-1}x + a_n$ has n distinct real roots x1, x2, . . . , xn, where $n > 1$. The polynomial $nx^{n-1} + (n-1)a_1x^{n-2} + (n-2)a_2x^{n-3} + \cdots + 2a_{n-2}x + a_{n-1}$ has roots y1, y2, . . . , yn−1. Prove that

$$\frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n} > \frac{y_1^2 + y_2^2 + \cdots + y_{n-1}^2}{n-1}.$$

3.一多項式 在n>1有n個不同實根x1,x2,…,xn。另一多項式 則有根y1,y2,…yn-1。證明 $\frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n} > \frac{y_1^2 + y_2^2 + \cdots + y_{n-1}^2}{n-1}.$

4. Each of Peter and Basil draws a convex quadrilateral with no parallel sides. The angles between a diagonal and the four sides of Peter's quadrilateral are α, α, β and γ in some order. The angles between a diagonal and the four sides of Basil's quadrilateral are alsoα, α, β and γ in some order. Prove that the acute angle between the diagonals of Peter's quadrilateral is equal to the acute angle between the diagonals of Basil's quadrilateral.

4. Peter 和 Basil 各畫一個沒有平行邊的凸四邊形。Peter的四邊形的對角線和四個邊之間的角度按某種順序為 α、α、β 和 γ。Basil四邊形的對角線與四個邊之間的角度按某種順序也為α、α、β和γ。證明Peter四邊形的對角線之間的銳角等於Basil四邊形的對角線之間的銳角

5. The positive integers are arranged in a row in some order, each occuring exactly once. Does there always exist an adjacent block of at least two numbers somewhere in this row such that the sum of the numbers in the block is a prime number?
5. 正整數按某種順序排列成一列，每個正整數恰好出現一次。在這一列的某處是否總是存在至少兩個數字的相鄰塊，使得區塊中的數字之和是質數？

6. Seated in a circle are 11 wizards. A different positive integer not exceeding 1000 is pasted onto the forehead of each. A wizard can see the numbers of the other 10, but not his own. Simultaneously, each wizard puts up either his left hand or his right hand. Then each declares the number on his forehead at the same time. Is there a strategy on which the wizards can agree beforehand, which allows each of them to make the correct declaration?
6. 11 個巫師圍坐成一圈。將不超過1000的不同正整數貼到每個人的額頭上。每個巫師可以看到其他 10 人的號碼，但看不到自己的。同時，每個巫師舉起左或右手，然後每個人同時宣告自己額頭上的數字。巫師們有沒有辦法事先弄一個方法，讓他們每個人都能做出正確的宣告？

7. Each of three lines cuts chords of equal lengths in two given circles. The points of intersection of these lines form a triangle. Prove that its circumcircle passes through the midpoint of the segment joining the centres of the circles.

7. 三線中的每一條線在兩個給定的圓中切割等長的弦。這些線的交點形成一個三角形。證明它的外接圓通過弓形連接圓心的線段的中點。

Note: The problems are worth 3+3, 6, 6, 7, 8, 8 and 8 points respectively.

相似題

A triangle has an angle of measure θ.
It is dissected into several triangles ,and all angles of the resulting triangles are less than θ.
What is the condition of θ?

三角形的量角為 θ。 它被分解成幾個三角形，在這些三角形的角都小於θ的情況下，那他的條件是甚麼?

在這裡θ一定會被分割成最少兩個角，分割後比較小的那個角最大為$\frac{\theta}{2}$，那他所屬的三角形中的最大角最小則是$\frac{180°-\frac{\theta}{2}}{2}$，且也要小於θ，故可以列出$\frac{180°-\frac{\theta}{2}}{2} < \theta$，$180° > \theta > 72°$

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