International Math Tournament
環球城市數學競賽

第五組

Senior Spring, A-Level

2012

組員

410731104林政勳

410731106王麒彰

410731107王霆軒

410731120許定閎

410731148許瑜芹

410931245梁靜玲

1.

Q:

　　在一個警衛隊中，每個警衛都被分配了一個不同的正整數。對於任意兩個守衛，分配給他們的兩個數字的比例至少為3:1。被分配了編號n的守衛則連續n天上班，連續n天休息，連續n天重新上班，且警衛不需要在同一天開始他們的職責。有沒有可能在任何一天，這樣的警衛隊伍中至少有一個人值班？

In a team of guards, each is assigned a different positive integer. For any two guards, the ratio of the two numbers assigned to them is at least 3:1. A guard assigned the number n is on duty for n days in a row, off duty for n days in a row, back on duty for n days in a row, and so on. The guards need not start their duties on the same day. Is it possible that on any day, at least one in such a team of guards is on duty?

1.

A:

設守衛為 $G_1, G_2, \dots, G_k$並給他們$n_1 > n_2$>…>$n_k \geq 1$的數字。事實上，對於 $n_i \geq 3n_{i+1}$，1≤ i < k 。 G_1間隔3n_2天不在值班。在此期間，有一個n_2 ≥3n_3天的子區間，在此期間G_2也不值班。重複這個參數直到到達G_k，則有一個區間n_k天沒有警衛值班。

Let the guards $G_1, G_2, \dots, G_k$ and let $n_1 > n_2$>…>$n_k \geq 1$ be the numbers assigned to them. In fact, $n_i \geq 3n_{i+1}$ for 1≤ i < k . There is an interval of 3n_2 days during which G_1 is not on duty. Within this interval, there is a subinterval of n_2 ≥3n_3 days during which G_2 is not on duty either. Repeating this argument until we reach G_k, we will have an interval of n_k days in which none of the guards are on duty.

1.

相似題

Q:

在一個部隊中，每個軍人都被分配了一個不同的正整數。
對於任意兩個軍人，分配給他們的兩個數字的比例至少為6:1，是要用來分配熬夜看守大門的。被分配了編號n的軍人則連續n天站崗，連續n天休息，連續n天重新站崗，且軍人不需要在同一天開始他們的職責。
有沒有可能在任何一天，這樣的部隊中至少有一個人值班？

1.

相似題

A:

設軍人為 $P_1, P_2, P_3 \cdots\cdots P_k$並給他們$P_1 > P_2 > P_3 \cdots\cdots > P_k \geq 1$的數字。事實上，對於 $P_i \geq 6P_{i+1}$，$1 \leq i < k$。 P_1間隔$6P_2$天不在看守。在此區間內，有一個$P_2 \geq 6P_3$天的子區間，在此期間P_2也不看守。重複這個參數直到到達P_k，則有一個區間P_k天沒有軍人看守。

2.

Q:

一百個點被標記在一個圓圈內，無三點共線。
證明：所有不重複的兩點連接後，這五十條線可以在圓內相互交叉。

One hundred points are marked inside a circle, with no three in a line. Prove that it is possible to connect the points in pairs such that all fifty lines intersect one another inside the circle.

3.

Q:

令n為正整數。證明：存在整數 $a_1, a_2, \dots, a_n$使得 對於任何整數x，存在一數$(\cdots(((x^2+a_1)^2+a_2)^2+\cdots)^2+a^{n-1})^2+a_n$ 可以被$2n-1$整除。

Let n be a positive integer. Prove that there exist integers $a_1, a_2, \dots, a_n$ such that for any integer x, the number is $(\cdots(((x^2+a_1)^2+a_2)^2+\cdots)^2+a^{n-1})^2+a_n$ divisible by $2n-1$.

4.

Q:

Alex在一個空心單元立方體的六個內表面上分別標記了一個點。然後他通過線段連接相鄰面上的任意兩個標記點。證明：這些標記的線段總長度至少為$6\sqrt{2}$。

Alex marked one point on each of the six interior faces of a hollow unit cube. Then he connected by strings any two marked points on adjacent faces. Prove that the total length of these strings is at least $6\sqrt{2}$.

5.

Q:

設l是一個三角形 ABC 內圓的切線。令l_a, l_b和l_c是各自的圖像l，在∠A, ∠B, ∠C的外角平分線上的映射下，證明由這些線組成的三角形與ABC全等。

Let l be a tangent to the incircle of triangle ABC. Let l a, l b and l c be the respective images of l under reflection across the exterior bisector of ∠A, ∠B and ∠C. Prove that the triangle formed by these lines is congruent to ABC.

6.

Q:

我們試圖用無限的矩形序列覆蓋平面，允許其重疊。

(a) 如果第n個矩形的面積是n^2，則對於每個n，是否總是可能成立？

(b) 如果每個矩形都是正方形，那是否總是可能成立，並且對於任何數 N，是否存在總面積大於 N 的正方形？

We attempt to cover the plane with an infinite sequence of rectangles, overlapping allowed.

(a) Is the task always possible if the area of the nth rectangle is n^2 for each n?

(b) Is the task always possible if each rectangle is a square, and for any number N, there exist squares with total area greater than N?

7.

Q:

Konstantin有一堆 100 塊鵝卵石。在每一步中，他選擇一堆並將其分成兩部分，直到他得到 100 堆，每堆只有一塊鵝卵石。

(a) 證明在某一時刻，有 30 堆石堆，總共正好有 60 顆鵝卵石。
(b) 證明在某一時刻，有 20 堆石頭，總共正好有 60 顆鵝卵石。
(c) 證明Konstantin可以以這樣一種方式進行，即在任何時候，有 19 堆，總共包含 60 顆鵝卵石。

Konstantin has a pile of 100 pebbles. In each move, he chooses a pile and splits it into two smaller ones until he gets 100 piles each with a single pebble.

(a) Prove that at some point, there are 30 piles containing a total of exactly 60 pebbles.
(b) Prove that at some point, there are 20 piles containing a total of exactly 60 pebbles.
(c) Prove that Konstantin may proceed in such a way that at no point, there are 19 piles containing a total of exactly 60 pebbles.