數學解題方法期中報告

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Prove that

$$\frac{1}{1999} < \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdots \frac{1997}{1998} < \frac{1}{44}$$

Let $P = \frac{1}{2} \cdot \frac{3}{4} \cdot \ldots \cdot \frac{1997}{1998}$

Since $\frac{1}{2} > \frac{1}{3}, \frac{3}{4} > \frac{3}{5}, \ldots, \frac{1997}{1998} > \frac{1997}{1999}$,

$P > \frac{1}{3} \cdot \frac{3}{5} \cdot \ldots \cdot \frac{1997}{1999} = \frac{1}{1999}$

Also , $\frac{1}{2} < \frac{2}{3}, \frac{3}{4} < \frac{4}{5}, \ldots, \frac{1997}{1998} < \frac{1998}{1999}$

Hence ,

$P < \frac{2}{3} \cdot \frac{4}{5} \cdot \ldots \cdot \frac{1998}{1999} = (\frac{2}{1} \cdot \frac{4}{3} \cdot \ldots \cdot \frac{1998}{1997}) \cdot \frac{1}{1999} = \frac{1}{P} \cdot \frac{1}{1999}$

$\Rightarrow P^2 < \frac{1}{1999} < \frac{1}{1936} = \frac{1}{44^2}$

$\Rightarrow \frac{1}{1999} < P = \frac{1}{2} \cdot \frac{3}{4} \cdot \ldots \cdot \frac{1997}{1998} < \frac{1}{44}$

【延伸題】

Prove that

$$\frac{1}{6667} < \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdots \frac{6665}{6666} < \frac{1}{81}$$

Let $P = \frac{1}{2} \cdot \frac{3}{4} \cdot \ldots \cdot \frac{6665}{6666}$

Since $\frac{1}{2} > \frac{1}{3}, \frac{3}{4} > \frac{3}{5}, \ldots, \frac{6665}{6666} > \frac{6665}{6667}$,

$P > \frac{1}{3} \cdot \frac{3}{5} \cdot \ldots \cdot \frac{6665}{6667} = \frac{1}{6667}$

Also , $\frac{1}{2} < \frac{2}{3}, \frac{3}{4} < \frac{4}{5}, \ldots, \frac{6665}{6666} < \frac{6666}{6667}$

Hence ,

$P < \frac{2}{3} \cdot \frac{4}{5} \cdot \ldots \cdot \frac{6666}{6667} = (\frac{2}{1} \cdot \frac{4}{3} \cdot \ldots \cdot \frac{6666}{6665}) \cdot \frac{1}{6667} = \frac{1}{P} \cdot \frac{1}{6667}$

$\Rightarrow P^2 < \frac{1}{6667} < \frac{1}{6561} = \frac{1}{81^2}$

$\Rightarrow \frac{1}{6667} < P = \frac{1}{2} \cdot \frac{3}{4} \cdot \ldots \cdot \frac{6665}{6666} < \frac{1}{81}$